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模糊α-cuts 法與蒙地卡羅法之鮭魚存活率不確定性分析比較

Comparison of Fuzzy α-cuts Method and Monte Carlo Method in Uncertainty Analysis of Salmonid Embryo Survival

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摘　要

本文針對礫石河床之鮭魚存活率進行不確定性分析。本研究採用模糊理論之 α-cuts 法探討模式參數之不確定性對鮭魚存活率之影響，並與蒙地卡羅模擬結果進行比較。研究結果顯示模糊理論 α-cuts 法因未考慮模式參數組合之機率，故計算所得之存活率分布範圍較廣，介於 13～91%間，最可能存活率為 80%。蒙地卡羅法模擬結果介於 30～90%間，最大相對頻率發生於存活率 82～83%間。本研究經由模式參數之相關性與敏感度分析結果得知產生最小存活率之參數組合屬於極不可能發生之情況，且為影響模式結果至鉅之最敏感組合，因此應加以刪除。根據此原則修改之結果具有較小之存活率分布範圍，並與蒙地卡羅模擬結果較為吻合。本研究所提出之改良式 α-cuts 法乃將模式參數之相關性與敏感度納入考量，不但可增進計算結果之精確度，更可提高不確定性分析之效率。

關鍵詞：不確定性分析，模式參數，模糊理論，蒙地卡羅模擬，相關性，敏感度。

ABSTRACT

This paper performs uncertainty analysis for salmonid embryo survival in spawning gravels. Fuzzy α-cuts method is employed to investigate the uncertainty of embryo survival associated with parameter errors, and the result is compared with that of Monte Carlo simulation. The result of α-cuts method reveals a wider spread than that obtained from Monte Carlo simulation because all combinations of parameter values are considered equally possible in fuzzy calculation. The results obtained using fuzzy calculation range from 13% to 91%, with the most likely survival rate of 80%. Embryo survival rate simulated by Monte Carlo method ranges from 30% to 90%, with the maximum relative frequency occurring between 82-83%. The correlation between parameters and sensitivity analyses indicate that the combination of parameters that results in the extreme survival rate is not only very unlikely to occur but also greatly affected by the parameter errors, and thus should be eliminated. The result obtained from this modified procedure has a much smaller range of survival rate for every level of possibility and coincides better with the result of Monte Carlo simulation. The modified procedure of fuzzy method proposed in this study is based on parameter correlation and model sensitivity, which can be more accurate and effective in uncertainty analysis.

Keywords: Uncertainty analysis, Model parameter, Fuzzy theory, Monte Carlo simulation, Correlation, Sensitivity.

1. Introduction

Gravel-bed streams provide suitable locations for salmonids to use as spawning and incubation habitat. Natural and anthropogenic environmental changes can degrade the quality of incubation habitat and thus adversely affect salmonid embryo survival. A predictive model has been developed for quantifying embryo survival in spawning gravels as a function of sediment deposition (Wu, 2000). The model sensitivity to individual parameter uncertainty and the combined effects of errors in multiple parameter values have been examined (Wu et al., 2001, accepted). Herein the results of sensitivity analysis are further incorporated with an investigation of the model uncertainty associated with input parameters. The present study employs the possibilistic (or fuzzy) approach to account for parameter uncertainty, and compares the results of possibilistic and Monte Carlo probabilistic approaches. A modified procedure of fuzzy method based on parameter correlation and model sensitivity is proposed to provide more realistic outcomes for the uncertainty of embryo survival.

2. Overview of Embryo Survival Model

Salmonid embryo survival model developed by Wu (2000) is chosen to perform the uncertainty analysis. The model consists of three governing equations, which clearly state the relationships between sediment deposit and substrate permeability, substrate permeability and apparent velocity, apparent velocity and embryo survival, respectively. It is proposed for the assessment of embryo survival in salmonid spawning gravel beds subject to fine-sediment deposition.

2.1 Relationship between sediment deposit and substrate permeability

Hydraulic resistance can be exerted on the flow through the accumulation of fine sediments in the voids of a porous medium. The mechanism is represented, with satisfactory results, by the following nonlinear relationship (Wu, 2000):

$$\frac{K}{K_0} = (4.54)\frac{(0.42-1.54\sigma)^3}{(0.58+1.54\sigma)^2} + (3.66)\left(\frac{d_s}{D_g}\right)^2 \sigma \quad (1)$$

in which K_0 and K represent the permeability of clean gravel bed, and reduced permeability resulting from sediment deposits; σ is the specific deposit, defined as (solid volume of sediment deposits)/(bulk volume of gravel bed including void space); D_g and d_s are the characteristic diameters of the gravel bed and the sediment deposits, for uniform materials they can be the median diameters, whereas for nonuniform particle sizes, D_{15} and d_{15} are recommended (Wu, 2000). For a clean gravel bed (when σ=0), the second term on the right is ineffective. While the first term vanishes when the pores are saturated with fine sediment (i.e., σ=0.42/1.54=0.273).

2.2 Relationship between substrate permeability and apparent velocity

Sediment-laden streamwater tends to flow through the spawning gravels from the high-pressure to the low-pressure region. A two-layer model is used to quantify this fine-sediment intrusion mode, and the apparent velocity through the two-layer redd gravels with surface flow across the bedding plane can be determined by

$$V' = \frac{(h/L_1)K_2}{(L_2/L_1)+(K_2/K_1)} \quad \text{.............................. (2)}$$

in which L_1 and L_2 are the length of flow path through layer 1 (sand seal) and layer 2 (surrounding gravel); K_1 and K_2 are the permeability of layer 1 and 2, respectively; and h is the total pressure head drop between the two regions. The ratio of K_1/K_2 used in (2) is simply the K/K_0 value calculated in (1) because σ represents the specific deposit in layer 1.

2.3 Relationship between apparent velocity and embryo survival

Apparent velocity is served as an indicator variable to quantify embryo survival in this model, and an empirical relationship between apparent

Table 1. Base values and ranges of model parameters

Parameter	Base value	Minimum	Maximum
σ	0.2	0.15	0.25
d_s/D_g	0.07	0.03	0.11
h/L_1	0.5	0.2	0.8
L_2/L_1	35	15	55

velocity and survival rate was developed through sets of experimental data (Wu, 2000):

$$S = -17.6(\log V') - 39.6(\log V') + 68.7 \quad \text{...........(3)}$$

in which S is percent survival and V' is the apparent velocity (in cm/s).

2.4 Parameter values

The four parameters considered in the uncertainty analysis are specific deposit σ, sediment-gravel size ratio d_s/D_g defined in (1), hydraulic pressure heads h/L_1, and intragravel flow path L_2/L_1 defined in (2), respectively. The base values of these parameters and their ranges are summarized in Table 1, which are selected based upon the data published in the original paper (Wu, 2000). The variable σ is set to the range between 0.15 and 0.25 because embryo survival is sensible to σ in this range.

3. Uncertainty Analysis

Due to the assumed simplification, and the sparse and imprecise nature of available information, assessment studies, such as the embryo survival model, are generally prone to considerable uncertainty. In this paper, a possibilistic approach is adopted to account for the model uncertainty associated with the parameter values, and then compared with the probabilistic approach.

3.1 Possibilistic approach

Fuzzy theory (Zadeh, 1965, 1978; Ross, 1997) is a powerful tool for treating the uncertainty

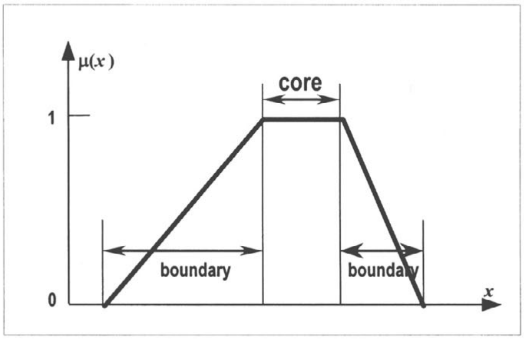

Figure 1 Core and boundaries of membership function

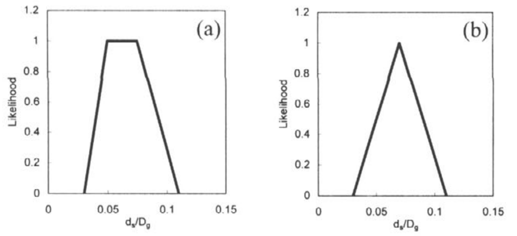

Figure 2 Typical membership function set: (a)Trapezoidal (b) Triangular

associated with vague and imprecise information. Instead of modeling parameter uncertainty with a PDF, the possibilistic approach uses fuzzy set. All information contained in a fuzzy set is described by its membership function $\mu(x)$ as shown in Figure 1, where x is the modeling parameter for uncertainty study. The core of the membership function comprises those elements whose $\mu(x)=1$, and the boundary of the membership function comprises those elements such that $0<\mu(x)<1$.

Figure 2 shows two example membership functions associated with d_s/D_g of the embryo survival model, which is considered as an uncertain parameter. The information conveyed in Figure 2(a) can be interpreted as the value of d_s/D_g is most likely between 0.05 and 0.075, no preference is given within this range; values lower than 0.03 and greater than 0.11 are considered impossible. Figure 2(b) is another typical shape of membership

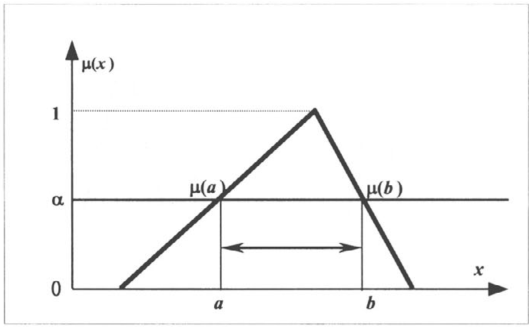

Figure 3 Illustration of α-cuts method

function, which can be interpreted as the value of d_s/D_g is most likely to be 0.07, any value falls out of the range [0.03,0.11] is considered impossible. Although any shape is possible, the selected membership function should be justified by the available information. The basic difference between PDF and fuzzy set is that the area below PDF equals unity, yet fuzzy set possesses no such property. Compared to PDF, fuzzy set is often preferable for representing information available from the field data, which is generally sparse. A rigorous definition of PDF requires a large number of measurements, which is hard to achieve in the field.

The fuzzy calculation is performed using the α-cuts method (Dubois and Prade, 1988) that has been applied by Guyonnet et al. (1999) among other scientists to compare with Monte Carlo method in risk assessment. For k parameters represented by fuzzy sets P_1 through P_k and the target model expressed as $f(P_1,....,P_k)$, the procedure of α-cuts method is summarized as follows:

(1)Set the level of possibility α of the membership function.

(2)For each fuzzy set P_1 through P_k, find the corresponding a and b values such that $\mu(a)=\mu(b)=\alpha$ (see Figure 3).

(3)Calculate the maximum and minimum values of $f(P_1,....,P_k)$ considering all the values located within [a, b] of each fuzzy set.

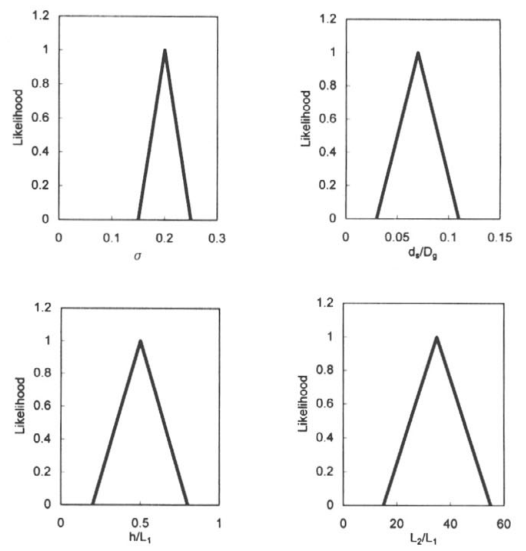

Figure 4 Uncertain parameters represented in Fuzzy sets

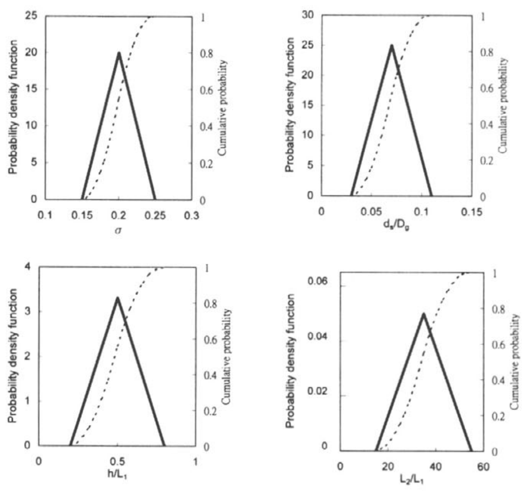

Figure 5 Uncertain parameters represented in PDFs and corresponding CDFs

(4)Apply the maximum and minimum values obtained from step 3 as the upper and lower limits to the α -cut of $f(P_1,....,P_k)$.

(5)Repeat steps 1 to 4 for another α level.

(6)Build the membership function of $f(P_1,....,P_k)$ with sets of maximum and minimum values calculated for each α -cut.

The embryo survival or $f(\sigma,\frac{d_s}{D_g},\frac{h}{L_1},\frac{L_2}{L_1})$ of the target model considered for uncertainty analysis is described in section 2. The base values and ranges of model parameters are listed in Table 1, and presented graphically in Figure 4 as fuzzy sets with triangular membership functions.

3.2 Probabilistic approach

The Monte Carlo simulation is carried out to compare with the fuzzy α -cuts method. The Monte Carlo simulation is performed by generating a sufficiently large number (herein 10,000 samples) of uncertain parameters that follow the PDFs shown in Figure 5. The shapes of the PDFs and the membership functions in Figure 4 are identical, however, the areas under all PDFs, i.e. the corresponding CDFs equal unity. The random parameters are then used to calculate the embryo survival rate through relationship (1), (2), and (3). In this study, the procedures are repeated for 10 runs, which result in 100,000 realizations of the physical-based model. This number of realizations is considered sufficient to obtain a reasonable relative frequency distribution of the embryo survival. Detailed description of the Monte Carlo simulation can be found elsewhere (e.g. Vose, 1996).

4. Results and Discussion

This section presents the results of uncertainty analysis. Results obtained using fuzzy and Monte Carlo calculations are first explored in section 4.1, and a comparison between the two is given in section 4.2. Also in section 4.2, the possibility for the combinations of parameter values is discussed, and a modification is suggested to the results obtained by fuzzy α -cuts method.

4.1 Fuzzy calculation and Monte Carlo simulation

Figure 6 shows the results obtained using

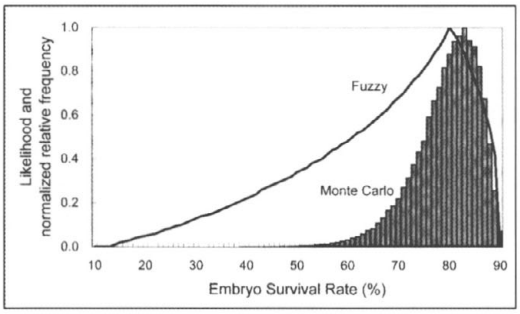

Figure 6 Comparison between Fuzzy and Monte Carlo simulation

fuzzy α -cuts method and Monte Carlo simulation. Fuzzy calculations were performed through 11 levels of possibility ranging from 0 to 1. For each α level, ranges of value were first located for all four parameters, and the calculations considering all combinations of parameter values within the range were performed to locate the minimum and maximum survival rates. Procedures were repeated for all levels of possibility.

Note that the curve in Figure 6 dropped abruptly at the right end towards 90%. The results obtained using fuzzy calculation range from 13% to 91% for the survival rate. In Figure 6, the most likely result of survival rate, i.e. the survival rate calculated at the level of possibility being equal to 1, is 80%. This result could be easily verified by applying the base values in Table 1 into (1) to (3). When the level of possibility decreases, the range of value for each parameter increases, therefore, the calculated survival rate spread wider. For example, the calculated results for survival rate range from 75% to 84% when the level of possibility drops to 0.8.

For a clearer comparison in Figure 6, the results of Monte Carlo simulation (i.e. the relative frequency distribution of survival rate) are normalized such that the maximum relative frequency is equal to 1. The embryo survival rates obtained by Monte Carlo method range between 30% and 90%, with a mean of 77%. The maximum relative frequency occurs for the survival rate between 82-83%.

Reading from Figure 6, we immediately notice the wider spread of the results obtained from fuzzy calculation compared with those obtained from Monte Carlo simulation. Guyonnet et al. (1999) pointed out that the fuzzy calculation is more conservative because it considers all combinations of parameter values are equally possible, regardless of their probabilities. On the contrary, in Monte Carlo simulation, the probability distribution was assigned to each parameter, and the combinations of parameter values with low probabilities have even less chance to be randomly chosen. Our results confirmed the conclusion of Guyonnet et al. (1999). To improve the results of fuzzy calculation, we develop a modified procedure for eliminating the combinations of parameters that are very unlikely to occur, as detailed subsequently.

4.2 Modified fuzzy calculation

Before drawing any conclusions from the outcomes of fuzzy calculation, it is necessary to investigate the possibility of every combination of parameter values from a physical perspective. For every level of possibility, one locates the range of value for each parameter and substitutes various combinations of parameter values into (1)-(3) to obtain the maximum and minimum values of survival rate. Examining the results of fuzzy calculation, we notice that for every level of possibility, the minimum survival rate is always generated by a particular combination of parameters, i.e. when d_s / D_g and h / L_1 are at their minimum values, and σ and L_2 / L_1 are at their maximum values. When such kind of combination is reversed, the maximum survival rate is generated for every level of possibility. This finding is confirmed with the original paper published by Wu (2000) on the embryo survival model that survival rate increases with greater d_s / D_g and h / L_1, but decreases with greater σ and L_2 / L_1. Yet it is questionable that all conditions contributed to the extreme survival rate would occur at the same time in real life.

To further examine the possibility of the extreme survival rate, correlation between parameters should be explored. With the information gathered from literature review, relationships between the parameters used for uncertainty analysis can be summarized as follows:

(1)Sediment deposits into gravel bed increase with grain size (Lisle, 1989), i.e. σ and d_s / D_g are positively correlated.

(2)Sediment deposits increase with hydraulic gradient across the gravel bed (Wu and Huang, 2000), i.e. σ and h / L_1 are positively correlated.

(3)The seal thickness L_1 increases with the sediment deposits (Lisle, 1989), i.e. σ and L_2 / L_1 are negatively correlated.

Applying the aforementioned correlation between parameters to the results of fuzzy calculation, we can take a second look at the possibility of the condition for the extreme survival rate. The reviews indicate that σ and L_2 / L_1 are negatively correlated, which makes the situation that both parameters are at their maximum values very unlikely to happen. The reviews also indicate that the relationships between σ and d_s / D_g, and σ and h / L_1 are both positively correlated, which makes the situation that parameter σ is at its maximum while parameters d_s / D_g and h / L_1 are both at their minimum also unlikely to occur. This argument helps us to reasonably conclude that the extreme survival rate obtained from the combination (i.e. maximum σ and L_2 / L_1, minimum d_s / D_g and h / L_1) is almost impossible to happen in real life.

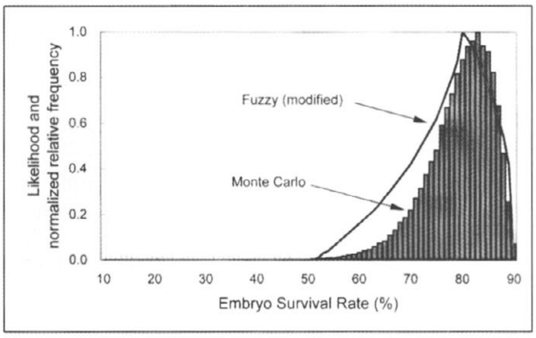

Figure 7 Comparison between Fuzzy (modified) and Monte Carlo simulation

The sensitivity analysis of embryo survival model (Wu et al., 2001, accepted) accompanying the present work is useful for further discussion. Recall that the errors in d_s / D_g cause the most serious effect on model results. The parameter d_s / D_g at its lower bound value has an even greater effect on survival rate. This indicates that the extreme survival rate generated by fuzzy method is not only very unlikely to occur but also greatly affected by the parameter errors. Accordingly, it is appropriate to suggest that the minimum survival rate generated with a particular combination of parameters, i.e. when σ and L_2 / L_1 are at their maximum values, and d_s / D_g and h / L_1 are at their minimum values, should be eliminated.

We continue to search for the violation of correlation between parameters and evaluate the sensitivity of model result to the errors in these parameters. It is thus suggested that the minimum survival rate generated with the combination that σ at its maximum value, and d_s / D_g, h / L_1, and L_2 / L_1 at their minimum values should also be eliminated. The reason for this elimination is that the violation of correlation involves two parameters d_s / D_g and h / L_1, which are the two most sensitive parameters in embryo survival model.

Figure 7 shows the modified result of fuzzy calculation. The modified result has a much smaller range for every level of possibility and coincides better with the result of Monte Carlo simulation. Since Monte Carlo simulation is computation intensive and time consuming, the modified procedure of fuzzy method based on parameter correlation and model sensitivity could be much more effective in analyzing uncertainty for ecological and environmental assessment models.

5. Summary and Conclusion

This study uses fuzzy α-cuts method and Monte Carlo simulation to perform uncertainty analysis on embryo survival model. The results obtained using fuzzy calculation range from 13% to 91%, with the most likely survival rate of 80%. Embryo survival rate simulated by Monte Carlo method ranges from 30% to 90%, with a mean of 77%. The maximum relative frequency occurs between 82-83%. The results obtained from fuzzy calculation reveal a wider spread than that obtained from Monte Carlo simulation because all combinations of parameter values are considered equally possible in fuzzy calculation. However, in Monte Carlo simulation the combinations of parameter values with low probabilities have even less chance to be randomly chosen. The correlation between parameters and sensitivity analyses indicate that the extreme survival rate obtained from the combination of maximum σ, L_2/L_1 and minimum d_s/D_g, h/L_1 is not only very unlikely to occur but also greatly affected by the parameter errors, and thus should be eliminated. The minimum survival rate generated with the combination that σ at its maximum value, and d_s/D_g, h/L_1, and L_2/L_1 at their minimum values should also be eliminated because the violation of correlation involves two most sensitive parameters, d_s/D_g and h/L_1. The result obtained from this modified procedure has a much smaller range of survival rate for every level of possibility and coincides better with the result of Monte Carlo simulation. Therefore, the modified procedure of fuzzy method based on parameter correlation and model sensitivity could be more effective in uncertainty analysis.

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