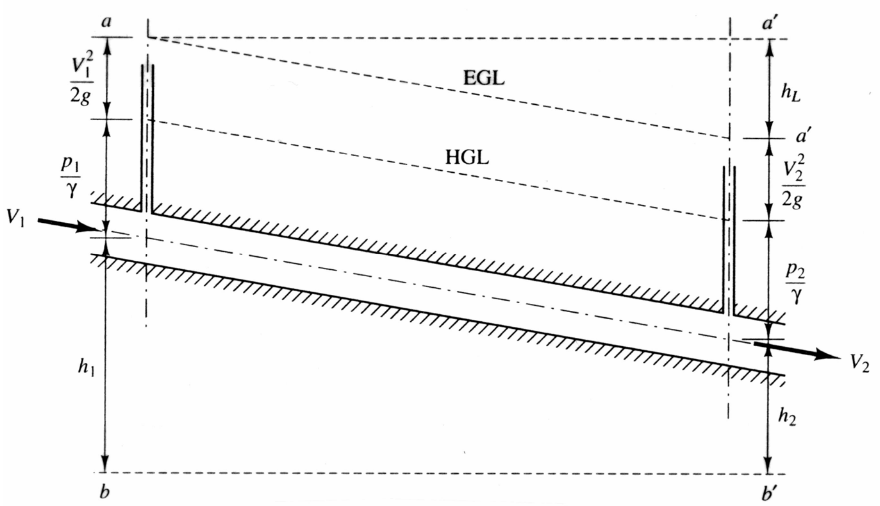
12. Pipe Flow

1. Energy Eqn for Pipe Flow (滿管)



$$z_1 + \frac{P_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} = z_2 + \frac{P_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + h_L$$

h_L之求法：

最常用 ①Darcy-Weisbach formula

$$h_L = f \cdot \frac{L}{D} \cdot \frac{V^2}{2g}$$

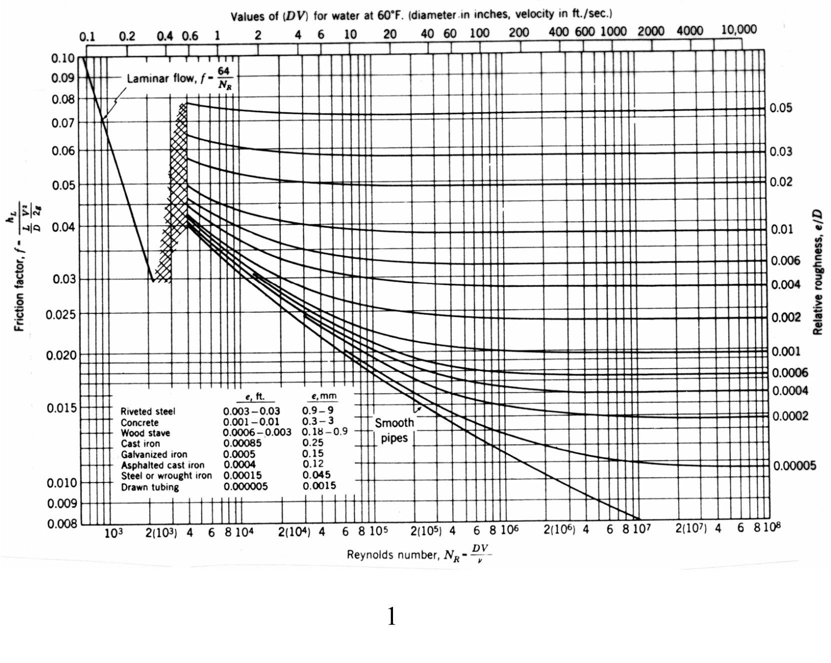
where L =Pipe length

D =Pipe diamater

f =Friction factor (dimensionless)

根據因次分析 $\Rightarrow \quad f = fct(e, V, D, \nu) = fct\left(\frac{e}{D}, \frac{VD}{\nu}\right)$

f 可由 Moody diagram (慕迪圖, p.348)查得



2. Minor Losses (次要水頭損失)

Reasons：①Entrance (入口)

②Exit (出口)

③Enlargement (Expansion) (斷面擴大)

④Contraction (斷面窄縮)

⑤Bends (彎管)

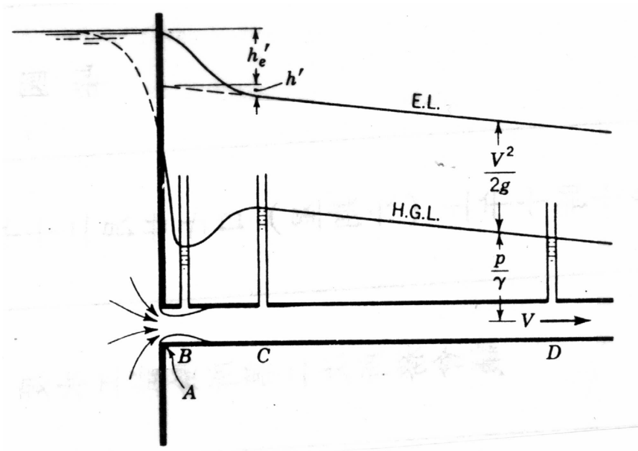
⑥Elbows (彎接頭)

⑦Valves (開關閥)

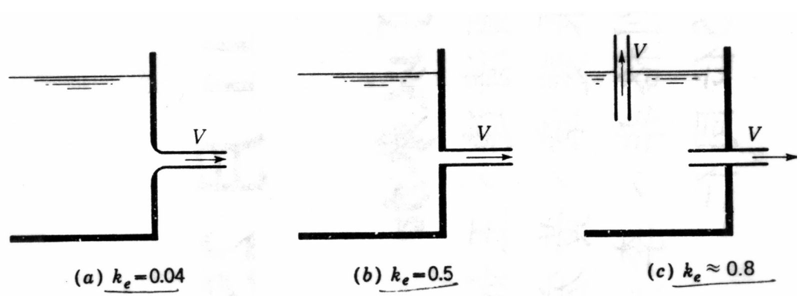
⑧Fittings (接管)

General form of minor losses：$h_L = k \cdot \frac{V^2}{2g}$

①Head Loss at Entrance (h_e)

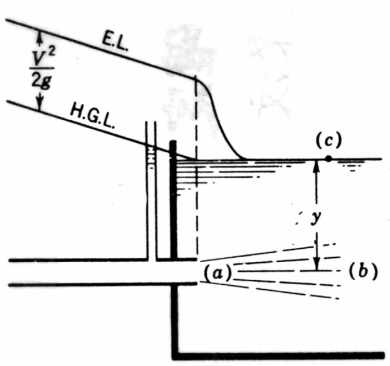


$$h_e = k_e \cdot \frac{V^2}{2g}$$



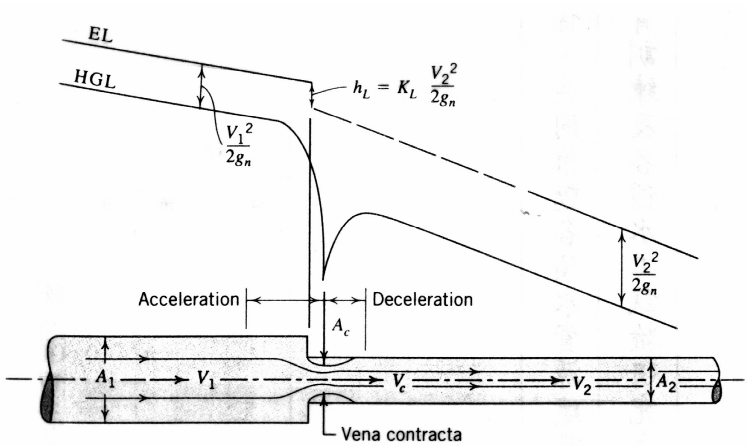
(a) $k_e = 0.04$ (b) $k_e = 0.5$ (c) $k_e \approx 0.8$

②Head Loss at Exit (h_x)



$$h_x = \frac{V^2}{2g}$$

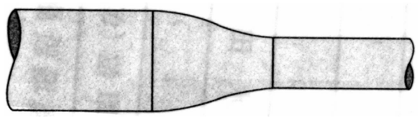
③Head Loss due to Contraction (h_c)



$$h_c = k_c \cdot \frac{V_2^2}{2g}$$

Abrupt contraction (突縮)

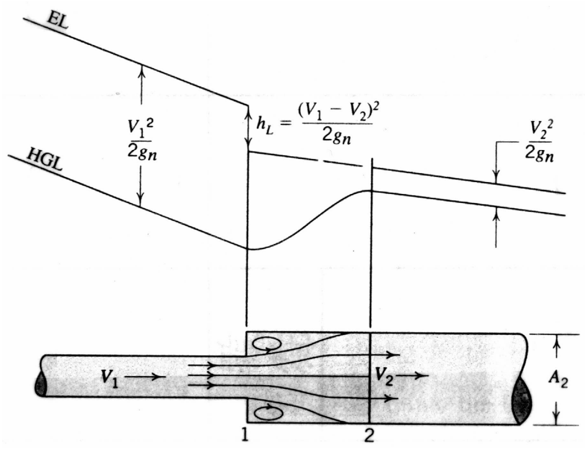
A_2 / A_1	0	0.2	0.4	0.6	0.8	1.0
k_c	0.5	0.41	0.30	0.18	0.06	0



Gradual contraction (緩縮)

$k_c \approx 0.05 \sim 0.10$

④Head Loss due to Enlargement (h_l)



$$h_l = \frac{(V_1 - V_2)^2}{2g} \qquad \Rightarrow \quad k_l = 1$$

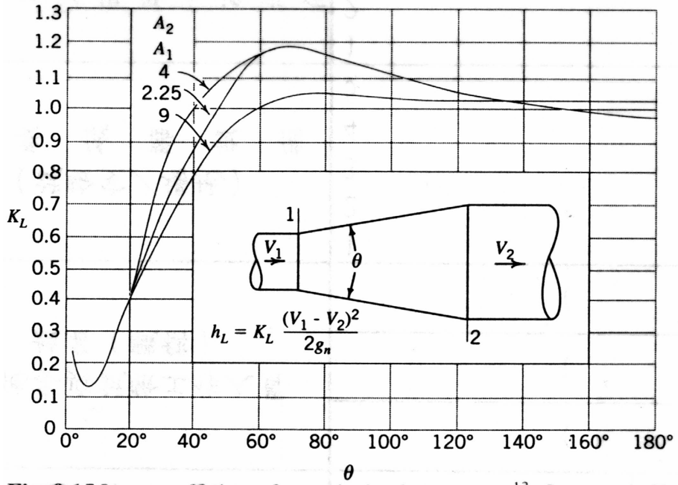


Fig. 9.15 Loss coefficients for conical enlargements.[13] (Source: A. H. Gibson, Hydraulics and its Applications, 4th ed., 1930.)

$$h_l = k_l \cdot \frac{(V_1 - V_2)^2}{2g}$$

⑤Head Loss at Bends (h_b)

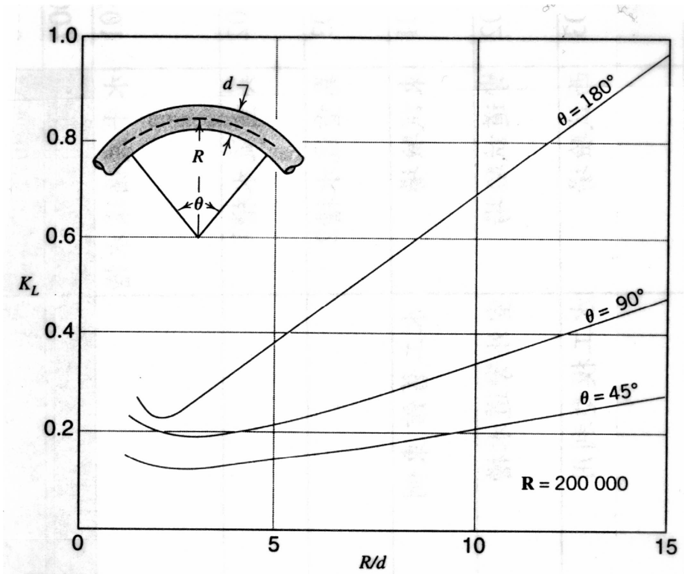


Fig. 9.20 Itō's loss coefficients for smooth bends (**R** = 200 000).

$$h_b = k_b \cdot \frac{V^2}{2g}$$

⑥Head Loss at Pipe Fittings (h_t)

$$h_t = k_t \cdot \frac{V^2}{2g}$$

Fitting	k_t
①Valve, wide-open	
Globe (球閥)	10
Angle (角閥)	2
Gate (門閥)	0.2
②Elbow (彎接頭)	
90°	1.5
45°	0.4
③Return bend (迴彎管)	1.5
④Tees (T分管)	2

- Total Head Loss (h_T)

$$h_T = h_e + h_x + h_L + h_c + h_l + h_b + h_t$$

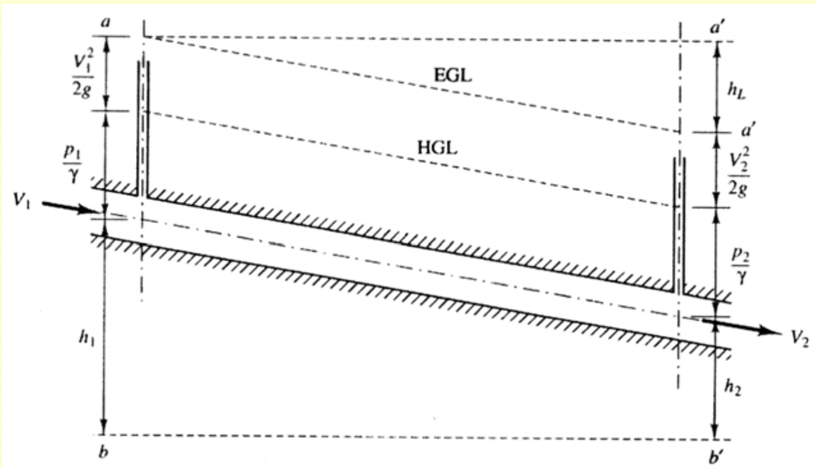
=入口＋出口＋管長＋窄縮＋擴張＋彎曲＋接頭

$$= \left(k_e + 1 + f \cdot \frac{L}{D} + k_b + k_t \right) \frac{V^2}{2g} + k_c \cdot \frac{V_2^2}{2g} + k_l \cdot \frac{(V_1 - V_2)^2}{2g}$$

12. Pipe Flow

管流之定義: 滿管 且受 壓力 及 重力 驅動而產生流動

1.Energy Equation for Pipe Flows



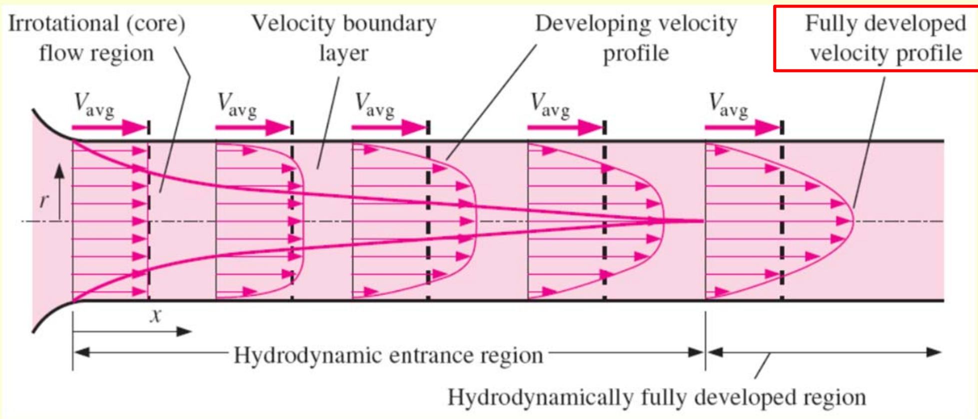
$$z_1+\frac{P_1}{\gamma}+\alpha_1\frac{V_1^2}{2g}=z_2+\frac{P_2}{\gamma}+\alpha_2\frac{V_2^2}{2g}+h_L$$

(h_L包括: 主要 及 次要 水頭損失)

2

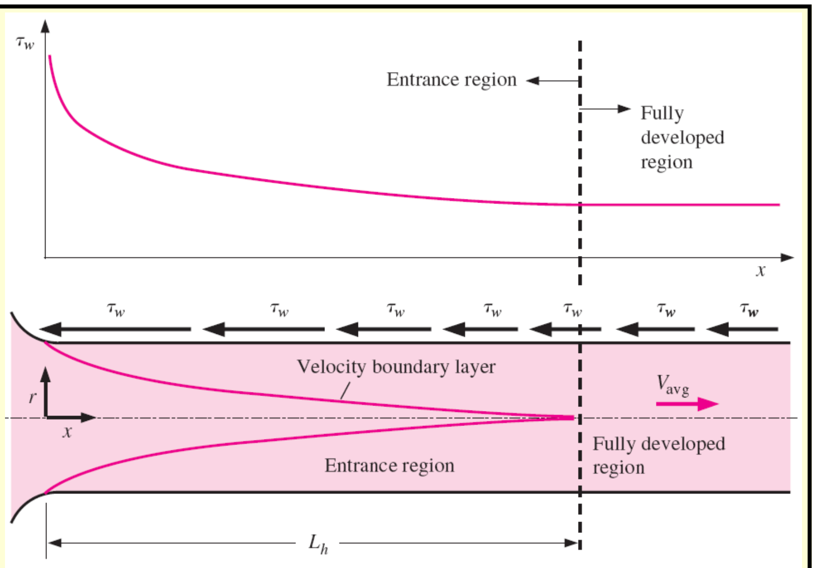
Development of pipe flow: *Entrance Region*

Velocity boundary layer
Boundary layer region
Irrotational (core) flow region



The development of the velocity boundary layer in a pipe.
The developed average velocity profile is parabolic in laminar flow, but somewhat flatter or fuller in turbulent flow.

Entry Length (入口長度)



The variation of wall shear stress in the flow direction for flow in a pipe from the entrance region into the fully developed region.

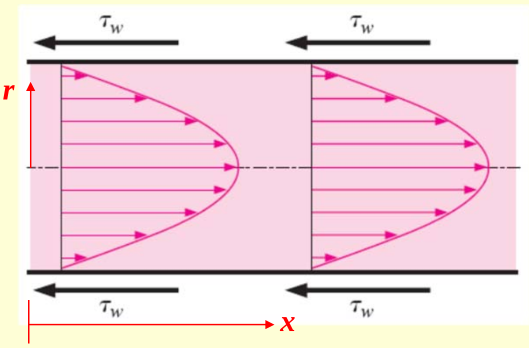
The hydrodynamic entry length is usually taken to be the distance from the pipe entrance to where the wall shear stress (and thus the friction factor) reaches within about 2 percent of the fully developed value.

Hydrodynamic entrance region: The region from the pipe inlet to the point at which the boundary layer merges at the centerline.

Hydrodynamic entry length L_h: The length of this region.

Hydrodynamically developing flow: Flow in the entrance region. This is the region where the velocity profile develops.

Hydrodynamically fully developed region: The region beyond the entrance region in which the velocity profile is fully developed and remains unchanged.



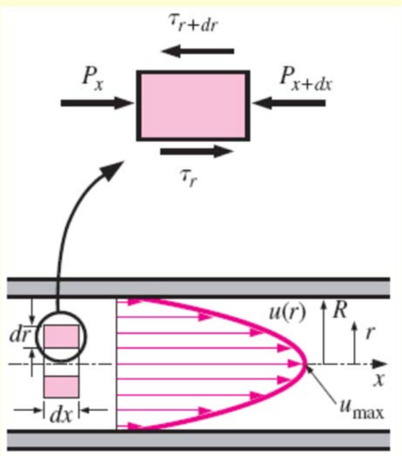
Hydrodynamically fully developed

$$\frac{\partial u(r,x)}{\partial x} = 0 \quad \rightarrow \quad u = u(r)$$

In the fully developed flow region of a pipe, the velocity profile does not change downstream, and thus the wall shear stress remains constant as well.

LAMINAR FLOW IN PIPES

$$(2\pi r\, dr\, P)_x - (2\pi r\, dr\, P)_{x+dx} + (2\pi r\, dx\, \tau)_r - (2\pi r\, dx\, \tau)_{r+dr} = 0$$

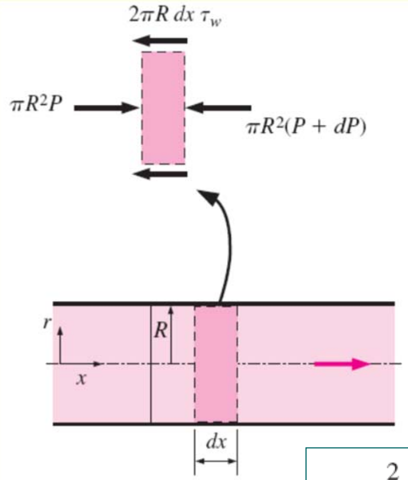


$$r\frac{P_{x+dx} - P_x}{dx} + \frac{(r\tau)_{r+dr} - (r\tau)_r}{dr} = 0$$

$$r\frac{dP}{dx} + \frac{d(r\tau)}{dr} = 0 \qquad \tau = -\mu(du/dr)$$

$$\frac{\mu}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right) = \frac{dP}{dx}$$

Free-body diagram of a ring-shaped differential fluid element of radius r, thickness dr, and length dx oriented coaxially with a horizontal pipe in fully developed laminar flow.



Force balance:

$$\pi R^2 P - \pi R^2(P + dP) - 2\pi R\,dx\,\tau_w = 0$$

Simplifying:

$$\frac{dP}{dx} = -\frac{2\tau_w}{R}$$

Free-body diagram of a fluid disk element of radius R and length dx in fully developed laminar flow in a horizontal pipe.

$$\frac{dP}{dx} = -\frac{2\tau_w}{R}$$ **(= constant)**

$$u(r) = \frac{r^2}{4\mu}\left(\frac{dP}{dx}\right) + C_1 \ln r + C_2$$

$\partial u/\partial r = 0$ at $r = 0$

$u = 0$ at $r = R$

Boundary conditions

$$u(r) = -\frac{R^2}{4\mu}\left(\frac{dP}{dx}\right)\left(1 - \frac{r^2}{R^2}\right)$$

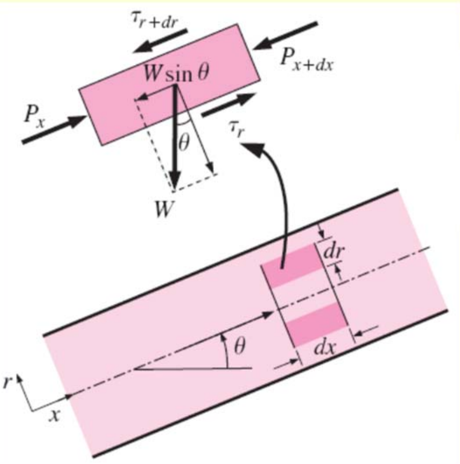
Average velocity

$$V_{avg} = \frac{2}{R^2}\int_0^R u(r) r\,dr = \frac{-2}{R^2}\int_0^R \frac{R^2}{4\mu}\left(\frac{dP}{dx}\right)\left(1 - \frac{r^2}{R^2}\right) r\,dr = -\frac{R^2}{8\mu}\left(\frac{dP}{dx}\right)$$

$$u(r) = 2V_{avg}\left(1 - \frac{r^2}{R^2}\right)$$ **Velocity profile**

$$u_{max} = 2V_{avg}$$ **Maximum velocity at centerline**

Effect of Gravity on Velocity in Laminar Flow



$$W_x = W \sin\theta = \rho g V_{element} \sin\theta = \rho g(2\pi r\,dr\,dx)\sin\theta$$

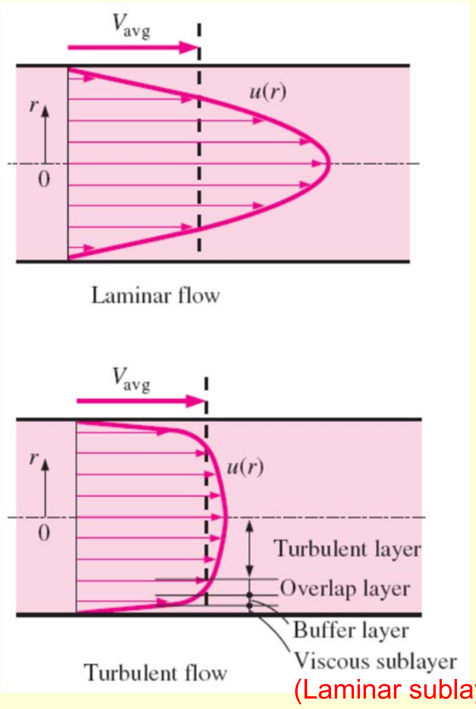
$$(2\pi r\,dr\,P)_x - (2\pi r\,dr\,P)_{x+dx} + (2\pi r\,dx\,\tau)_r - (2\pi r\,dx\,\tau)_{r+dr} - \rho g(2\pi r\,dr\,dx)\sin\theta = 0$$

$$\frac{\mu}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right) = \frac{dP}{dx} + \rho g \sin\theta$$

$$u(r) = -\frac{R^2}{4\mu}\left(\frac{dP}{dx} + \rho g \sin\theta\right)\left(1 - \frac{r^2}{R^2}\right)$$

Free-body diagram of a ring-shaped differential fluid element of radius *r*, thickness *dr*, and length *dx* oriented coaxially with an inclined pipe in fully developed laminar flow.

Fully Developed Velocity Profiles: Laminar & Turbulent Flows



(Laminar sublayer 層流次層)

(1) The very thin layer next to the wall where viscous effects are dominant is the **viscous** (or **laminar** or **linear** or **wall**) sublayer.

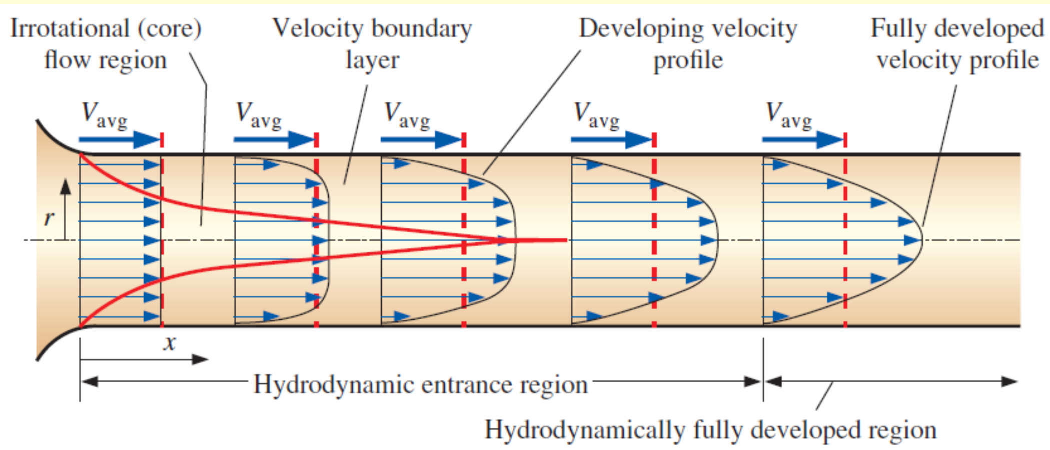
The velocity profile in this layer is very nearly *linear*, and the flow is streamlined.

(2) Next to the viscous sublayer is the **buffer layer**, in which turbulent effects are becoming significant, but the flow is still dominated by viscous effects.

(3) Above the buffer layer is the **overlap** (or **transition**) **layer**, also called the **inertial sublayer**, in which the turbulent effects are much more significant, but still not dominant.

(4) Above that is the **outer** (or **turbulent**) **layer** in the remaining part of the flow in which turbulent effects dominate over molecular diffusion (viscous) effects.

Entry Length (入口長度) for *Turbulent Flow*



The entry length is *much shorter* in turbulent flow.

Entry Lengths

The hydrodynamic entry length is usually taken to be the distance from the pipe entrance to where the wall shear stress (and thus the friction factor) reaches within about 2 percent of the fully developed value.

$$\frac{L_{h,\,\text{laminar}}}{D} \cong 0.05\text{Re}$$ hydrodynamic entry length for laminar flow

$$\frac{L_{h,\,\text{turbulent}}}{D} = 1.359\text{Re}^{1/4}$$ hydrodynamic entry length for turbulent flow

$$\frac{L_{h,\,\text{turbulent}}}{D} \approx 10$$ hydrodynamic entry length for turbulent flow, an approximation

The pipes used in practice are usually several times the length of the entrance region, and thus the flow through the pipes is often assumed to be fully developed for the entire length of the pipe.

This simplistic approach gives *reasonable* results for long pipes but sometimes poor results for short ones since it underpredicts the wall shear stress and thus the friction factor.

$$z_1 + \frac{P_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} = z_2 + \frac{P_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + h_L$$ **(h_L包括：主要及次要水頭損失)**

1. Major head loss (管流主要水頭損失：摩擦水頭損失 h_f)

h_f之求法： 最常用 **Darcy-Weisbach formula**

$$h_f = f \cdot \frac{L}{D} \cdot \frac{V^2}{2g}$$

where L = Pipe length

D = Pipe diameter

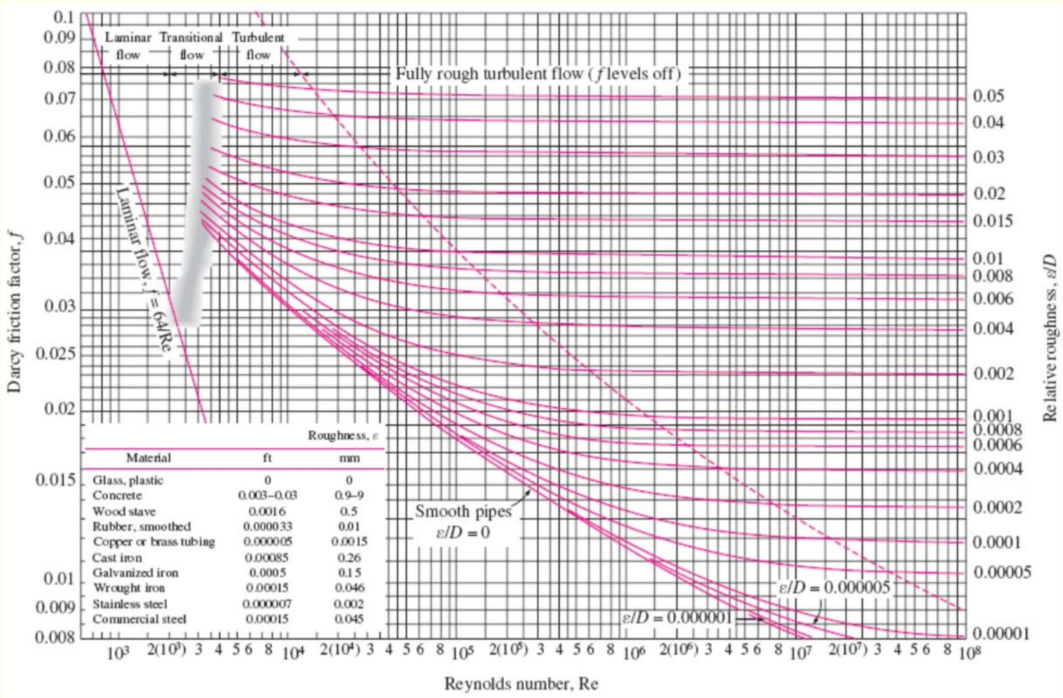
f = Friction factor (dimensionless)

根據因次分析 **(n – k = 5 - 2 = 3)** $\Rightarrow$ $f = fct(e, V, D, \nu) = fct\left(\frac{e}{D}, \frac{VD}{\nu}\right)$

f 可由 **Moody Chart** (慕迪圖) 查得

Roughness height (粗糙高度)

12



	Roughness, ε	
Material	ft	mm
Glass, plastic	0	0
Concrete	0.003–0.03	0.9–9
Wood stave	0.0016	0.5
Rubber, smoothed	0.000033	0.01
Copper or brass tubing	0.000005	0.0015
Cast iron	0.00085	0.26
Galvanized iron	0.0005	0.15
Wrought iron	0.00015	0.046
Stainless steel	0.000007	0.002
Commercial steel	0.00015	0.045

Moody Chart (慕迪圖)

2. Minor Head Losses (管流次要水頭損失)

Including : **(1) Entrance (管入口)**

(2) Exit (管出口)

(3) Expansion (管斷面擴大)

(4) Contraction (管斷面窄縮)

(5) Bend (彎曲管)

(6) Elbow (彎接頭)

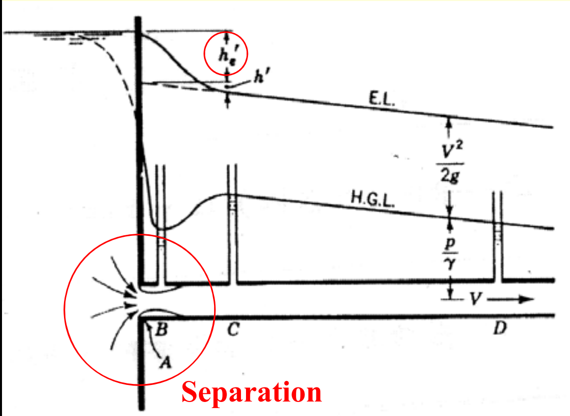
(7) Valve (管閥門)

(8) Fittings (管配件)

General form of minor losses：

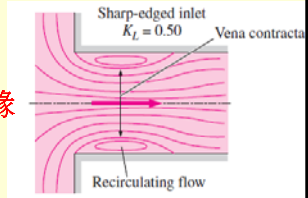
$$h_L = k \cdot \frac{V^2}{2g}$$

① Head Loss at Entrance (h_e)

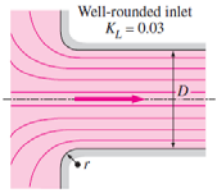


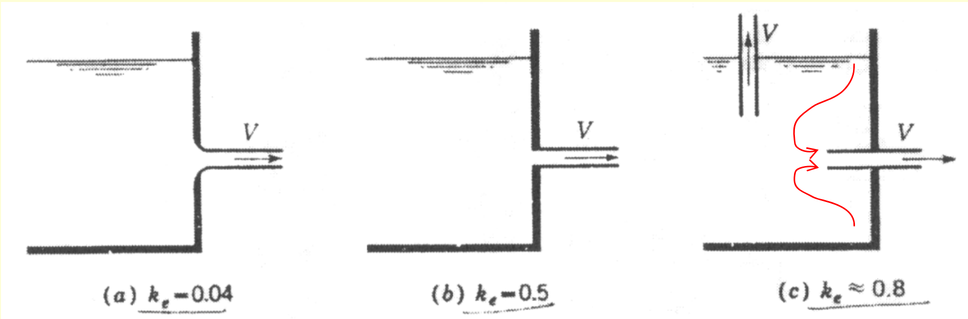
$$h_e = k_e \cdot \frac{V^2}{2g}$$

尖銳邊緣

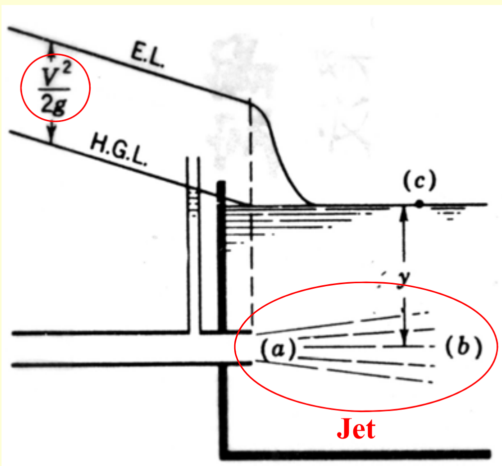


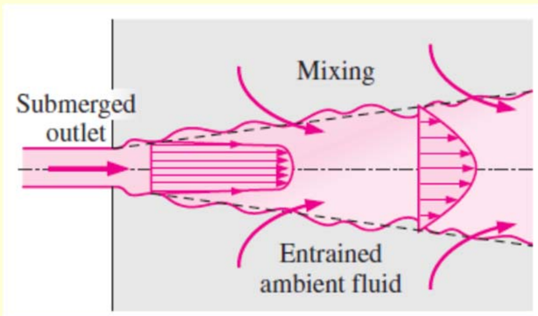
圓滑邊緣





② Head Loss at Exit (h_x)

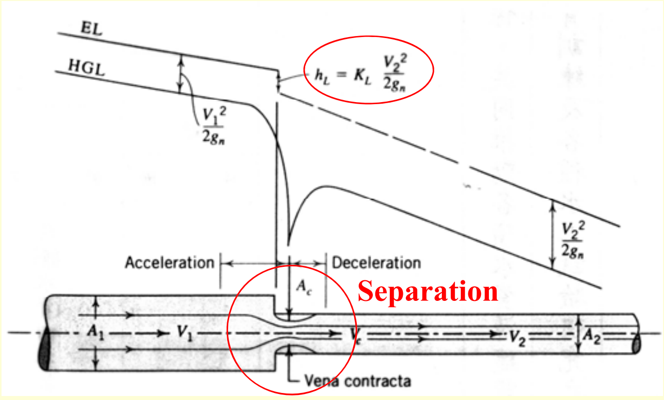




$$h_x = \frac{V^2}{2g}$$

- All the kinetic energy of the flow is "lost" (i.e., turned into thermal energy) through friction
- as the jet decelerates and mixes with ambient fluid downstream of a submerged outlet.

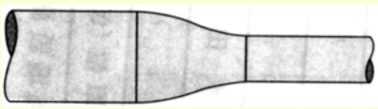
③ Head Loss due to Contraction (h_c)



$$h_c = k_c \cdot \frac{V_2^2}{2g}$$

Abrupt contraction (突縮)

A_2/A_1	0 ←	0.2	0.4	0.6	0.8	1.0
k_c	0.5	0.41	0.30	0.18	0.06	0

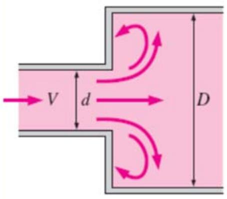


Gradual contraction (緩縮)

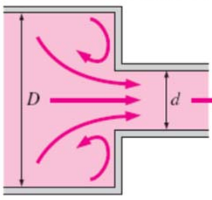
$k_c \approx 0.05 \sim 0.10$

Sudden Expansion and Contraction (based on the velocity in the smaller-diameter pipe)

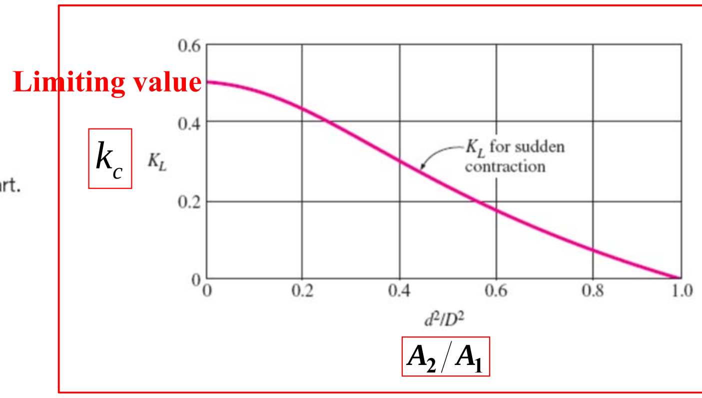
Sudden expansion: $K_L = \alpha\left(1 - \frac{d^2}{D^2}\right)^2$



Sudden contraction: See chart.

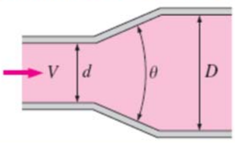


突縮



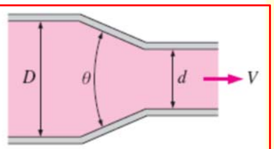
Gradual Expansion and Contraction (based on the velocity in the smaller-diameter pipe)

Expansion (for $\theta = 20°$):
$K_L = 0.30$ for $d/D = 0.2$
$K_L = 0.25$ for $d/D = 0.4$
$K_L = 0.15$ for $d/D = 0.6$
$K_L = 0.10$ for $d/D = 0.8$

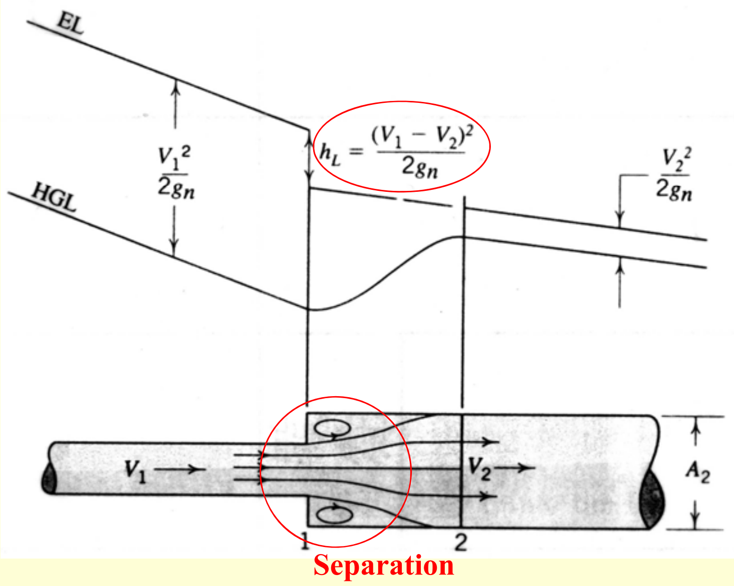


Contraction:
$K_L = 0.02$ for $\theta = 30°$
$K_L = 0.04$ for $\theta = 45°$
$K_L = 0.07$ for $\theta = 60°$

緩縮



④ Head Loss due to Expansion (h_l)



$$h_l = \frac{(V_1 - V_2)^2}{2g} \Rightarrow k_l = 1$$

⑤ Head Loss at Bend (h_b)

$$h_b = k_b \cdot \frac{V^2}{2g}$$

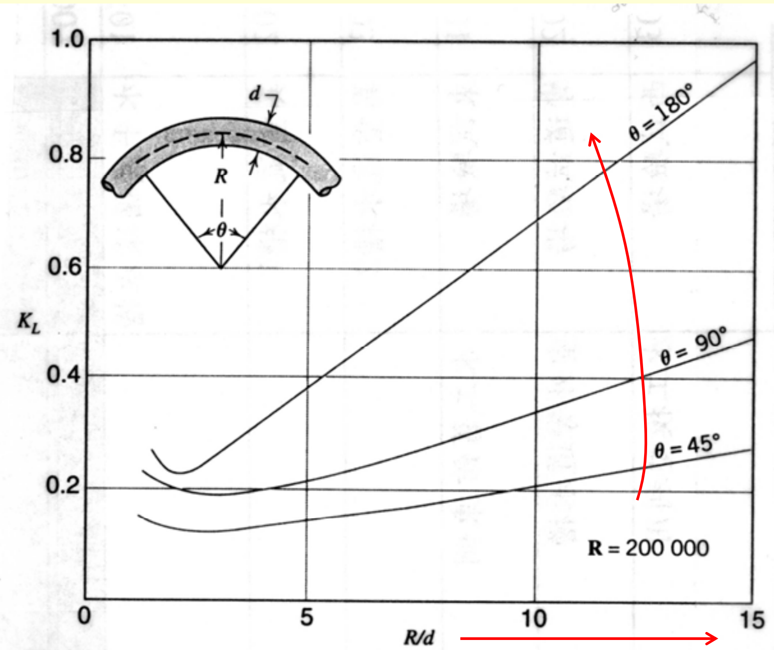


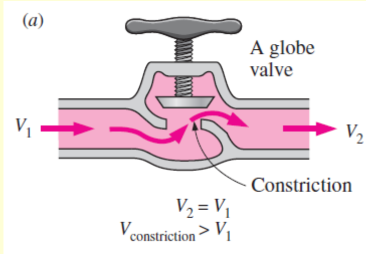
Fig. 9.20 Itō's loss coefficients for smooth bends (**R** = 200 000).

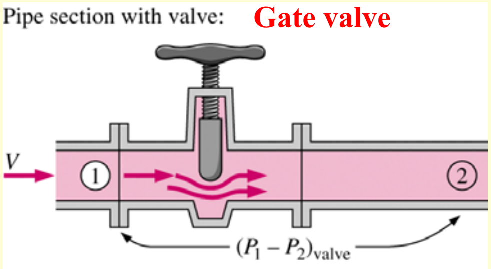
⑥ Head Loss at Pipe Fittings (h_t)

Fitting	
Valve (wide-open)	k_t
Globe (球閥)	10
Angle (角閥)	2
Gate (門閥)	0.2
Elbow (彎接頭)	
90°	1.5
45°	0.4
Return bend (U形迴彎管)	1.5
Tee (T分管)	2

$$h_t = k_t \cdot \frac{V^2}{2g}$$

Globe valve





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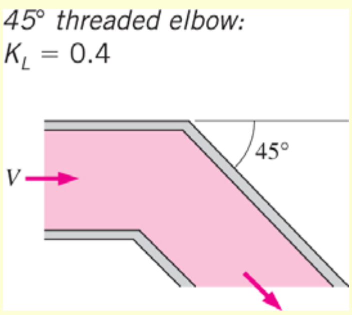
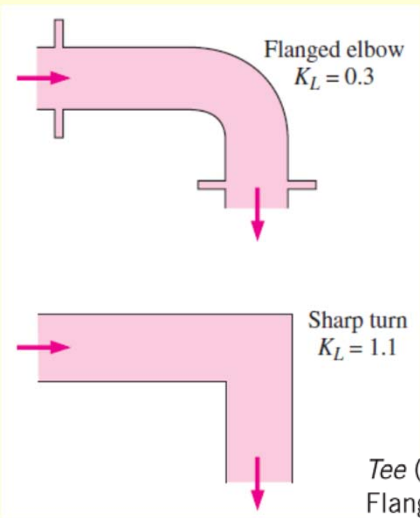
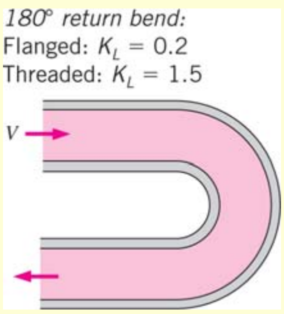
Elbow (彎接頭)

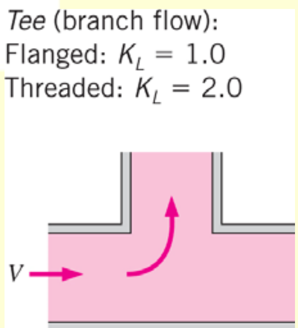






Return bend (U形迴彎管)

Tee (T分管)



- **Total Head Loss (h_T)**

$$h_T = h_e + h_x + \boxed{h_f} + h_b + h_t + h_c + h_l$$

=入口＋出口＋管摩擦＋彎曲＋配件＋窄縮＋擴張

$$= \left(k_e + 1 + \boxed{f \cdot \frac{L}{D}} + k_b + k_t \right) \frac{V^2}{2g} + k_c \cdot \frac{V_2^2}{2g} + k_l \cdot \frac{(V_1 - V_2)^2}{2g}$$

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