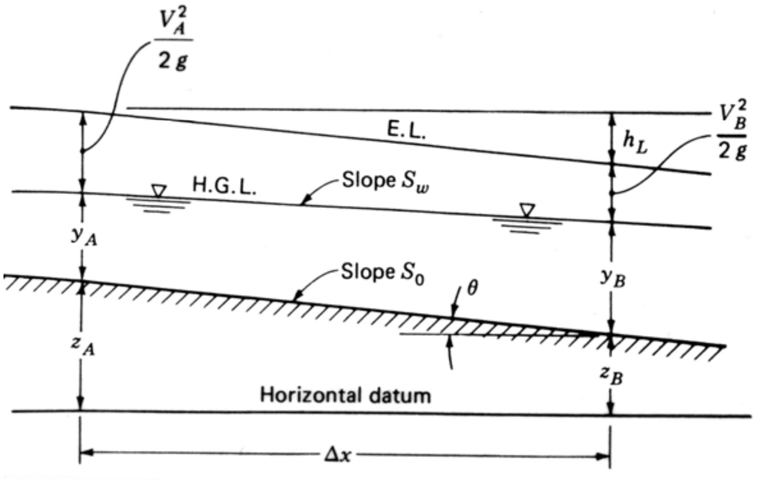
11. Open Channel Flow

1. Fundamentals

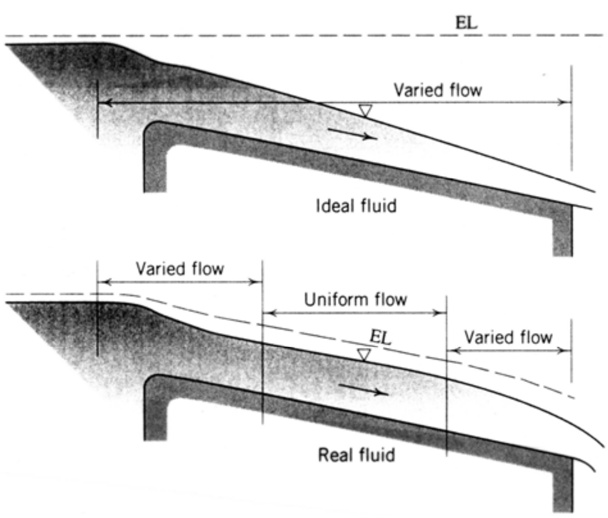
- Hydraulic Grade Line, Energy Line, and Head Loss



Energy eqn. between A and B：

$$z_A + y_A + \alpha_A \frac{V_A^2}{2g} = z_B + y_B + \alpha_B \frac{V_B^2}{2g} + h_L$$

- Uniform Flow vs. Varied Flow (Nonuniform Flow)



2. Uniform Flow（均勻流）

- Definition

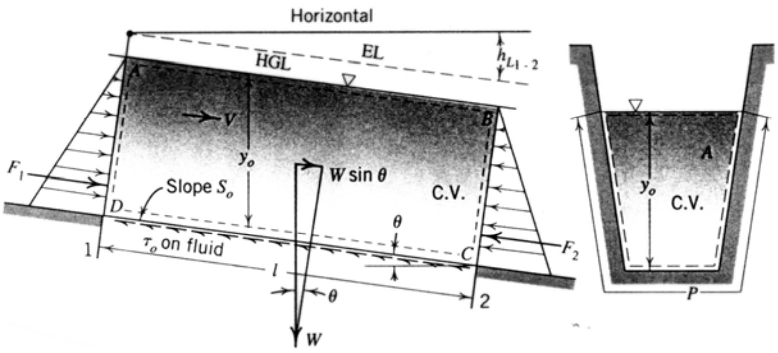
$$\begin{cases} y_A = y_B \\ V_A = V_B \\ S_0 = S_w = S_f = S \end{cases} \Rightarrow \quad h_L = z_A - z_B$$

式中 S_0 ＝底床坡降

S_w ＝水面坡降

S_f ＝能量坡降 $= h_L / \Delta x$

- Momentum Eqn



$$F_x = F_1 - F_2 + W \cdot \sin\theta - P \cdot \Delta x \cdot \tau = \beta \rho Q(V - V) = 0$$

$$W \cdot \sin\theta = P \cdot \Delta x \cdot \tau$$

$$(A \cdot \Delta x \cdot \gamma) \cdot S = P \cdot \Delta x \cdot \tau$$

$$\therefore \quad \boxed{\tau = \gamma \cdot \frac{A}{P} \cdot S = \gamma RS}$$

式中 $P =$ Wetted perimeter（潤周）

$R = \dfrac{A}{P} =$ Hydraulic radius（水力半徑）

- Empirical Formulas for Average Velocity

① Chezy eqn.

$$V = C\sqrt{RS}$$

式中 $C =$ Chezy's resistance factor

② Manning eqn.

$$V = \frac{1}{n} R^{2/3} S^{1/2} \qquad \text{（公制）}$$

$$V = \frac{1.49}{n} R^{2/3} S^{1/2} \qquad \text{（英制）}$$

式中　$n =$ Manning's roughness coefficient

(Manning's *n* given in Table 5, p.438-439)

● Uniform Flow Depth (or Normal Depth) 正常水深

When uniform flow → Normal depth (y_n)

Normal depth (y_n) → Obtained by Manning eqn through <u>trial-and-error</u>（試誤法）

3. Best Hydraulic Cross Section（最佳水力斷面）

(or Most Efficient Cross Section)

● 最佳水力斷面

若渠道坡度、糙度及通水面積A固定，當潤周P為最小值時，水力半徑R = A/P為最大值，此斷面輸水能力最大(Q 最大)，故為最佳水力斷面。

<u>Example</u>: 矩形斷面之最佳水力設計

$$R = \frac{By}{B+2y} = \frac{A}{(A/y)+2y} = \frac{Ay}{(A+2y^2)}$$

When $\frac{dR}{dy} = 0 \quad \rightarrow \quad R_{\max}$

$$\frac{d\left(\frac{Ay}{A+2y^2}\right)}{dy} = \frac{A(A-2y^2)}{(A+2y^2)^2} = 0$$

$A = 0$ (不合) 或 $A = 2y^2 = By$

$\therefore \quad B = 2y$ 時，$R_{\max} = y/2$

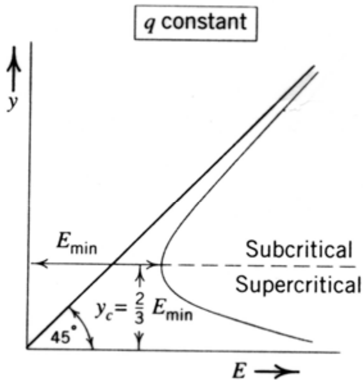
4. Specific Energy（比能），E

$$E = y + \frac{V^2}{2g} = y + \frac{1}{2g}\left(\frac{q}{y}\right)^2$$

式中 $$q = \frac{Q}{B} = \frac{B \cdot y \cdot V}{B} = y \cdot V$$

(for wide rectangular channel)

● Specific Energy Diagram（比能圖）



Specific energy diagram

When $$\frac{dE}{dy} = 1 - \frac{q^2}{gy^3} = 0$$

$$\rightarrow q^2 = y^2V^2 = gy^3$$

$$\rightarrow V^2 = gy \quad \Rightarrow \frac{V}{\sqrt{gy}} = Fr = 1 \qquad \text{(Critical flow)}$$

$$\rightarrow E_{min} = y + \frac{1}{2g}\left(\frac{gy^3}{y^2}\right) = \frac{3}{2}y$$

∴ Critical depth $$y_c = \frac{2}{3}E_{min} = \sqrt[3]{\frac{q^2}{g}}$$

$y > y_c \Rightarrow Fr < 1$ → Subcritical flow（亞臨界流）

$y < y_c \Rightarrow Fr > 1$ → Supercritical flow（超臨界流）

● Application of Specific Energy Diagram

① Flow over a bump（凸起物）

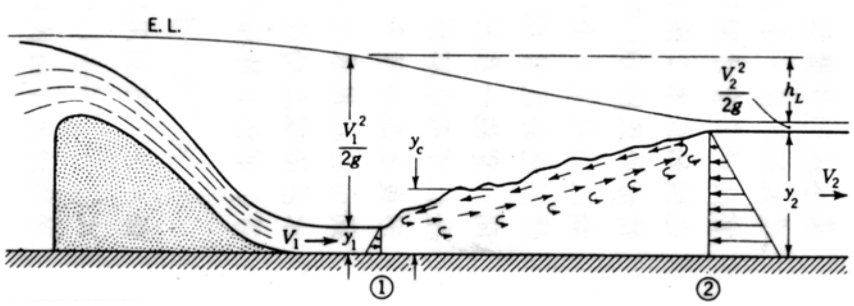
② Channel contraction（窄縮）

5. Hydraulic Jump（水躍）

- Supercritical flow → Subcritical flow

  ⇒ Hydraulic Jump

- Conjugate depth（共軛水深）



Momentum eqn: $F_x = F_1 - F_2 = \rho\, q(V_2 - V_1)$

in which $q = y_1 V_1 = y_2 V_2 \quad \Rightarrow \quad V_1 = \frac{q}{y_1}, V_2 = \frac{q}{y_2}$

$$F_1 = \frac{\gamma}{2} y_1^2, \quad F_2 = \frac{\gamma}{2} y_2^2$$

$$\Rightarrow \frac{\gamma}{2} y_1^2 - \frac{\gamma}{2} y_2^2 = \rho\, q^2 \left(\frac{1}{y_2} - \frac{1}{y_1} \right)$$

Solving this eqn yields:

$$\frac{y_2}{y_1} = \frac{1}{2}\left[-1 + \sqrt{1 + \frac{8q^2}{g y_1^3}} \right]$$

$$= \frac{1}{2}\left[-1 + \sqrt{1 + 8 \cdot \frac{V_1^2}{g y_1}} \right]$$

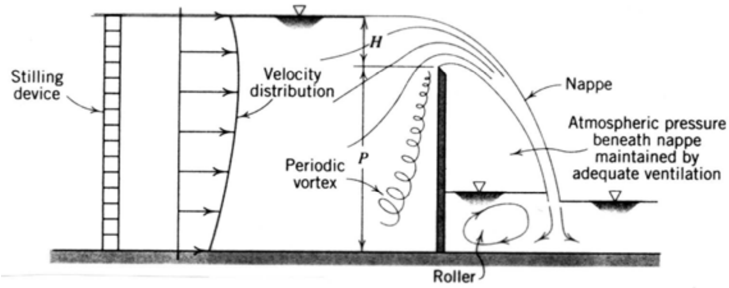
$$= \frac{1}{2}\left[-1 + \sqrt{1 + 8 Fr_1^2} \right]$$

in which $\quad Fr_1 = \frac{V_1}{\sqrt{g y_1}}$

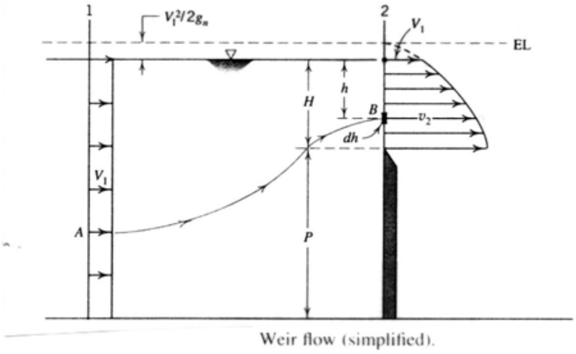
6. Measurement of Open-channel Flow

- Sharp-crested Weir（銳緣堰）

① Rectangular weir（矩形堰）



Weir flow (actual).



Weir flow (simplified).

$$H + 0 + \frac{V_1^2}{2g} = (H - h) + 0 + \frac{v_2^2}{2g}$$

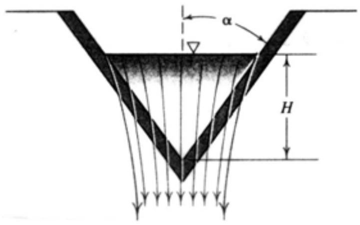
$$v_2 \cong \sqrt{2gh}$$

$$Q = \int v_2 dA = \int_0^H \sqrt{2gh} \cdot B \cdot dh = \frac{2}{3} \cdot \sqrt{2g} \cdot B \cdot H^{3/2}$$

$$\therefore \quad Q = C_W \cdot \frac{2}{3} \cdot \sqrt{2g} \cdot B \cdot H^{3/2}$$

② Triangular weir（三角堰）

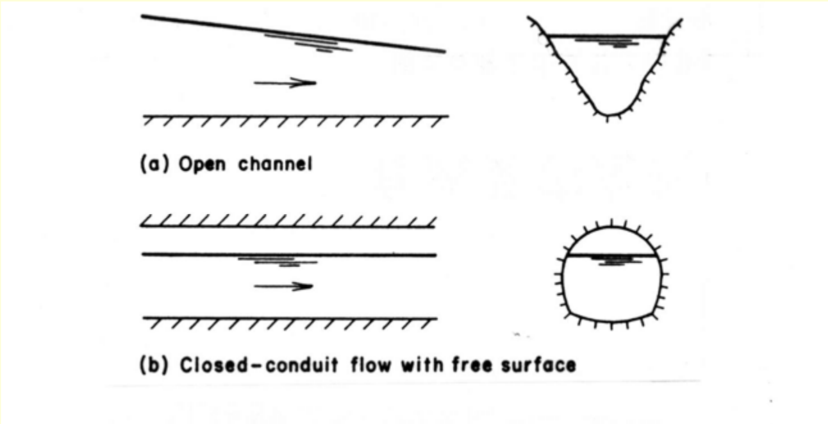
$$Q = C_W \cdot \frac{8}{15} \cdot \tan\alpha \cdot \sqrt{2g} \cdot H^{5/2}$$



11. Open Channel Flow

Definition

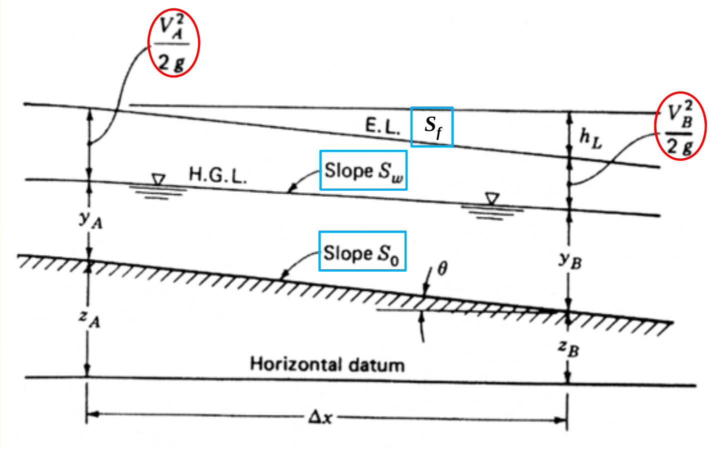
- **Open channel flow** (明渠流)，又稱 **Free-surface flow** (自由液面流)。
  於固體邊界內具有自由液面 **(free surface)** 之液體 受重力而流動者。



1

1. Fundamentals

● **Hydraulic Grade Line (HGL), Energy Line (EL), and Head Loss**



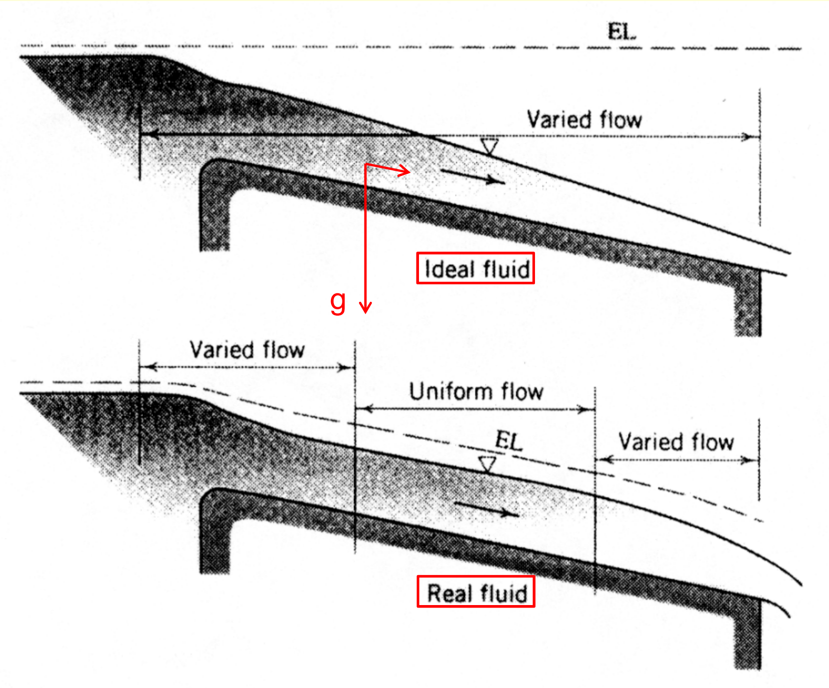
If $S_0 = S_w = S_f$

$\Rightarrow$ Uniform flow

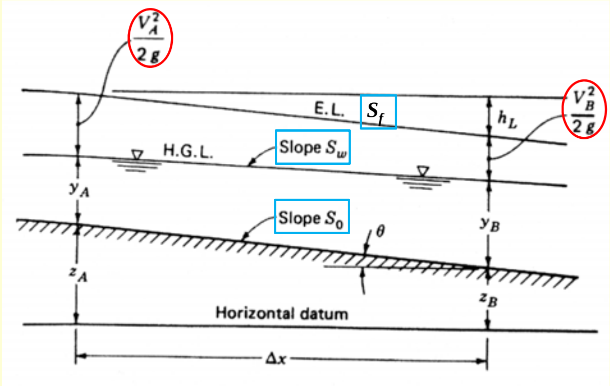
Energy eqn. between Section *A* and Section *B*：

$$z_A + y_A + \alpha_A \frac{V_A^2}{2g} = z_B + y_B + \alpha_B \frac{V_B^2}{2g} + h_L$$

● **Uniform Flow (等速流) vs. Non-uniform (or Varied) Flow (變速流)**



2. Uniform Flow (等速流)



- **Definition**

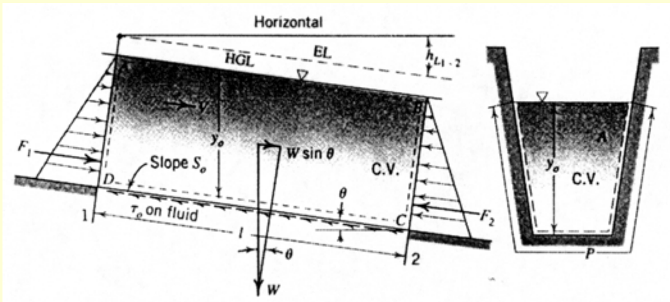
$$\begin{cases} y_A = y_B \\ V_A = V_B \\ S_0 = S_w = S_f = S \end{cases} \Rightarrow \quad h_L = z_A - z_B$$

式中 S_0 = 底床坡降

S_w = 水面坡降

S_f = 能量坡降 $= h_L / \Delta x$

- **Momentum Eqn for Uniform Flow**



$$F_x = \cancel{F_1} - \cancel{F_2} + W \cdot \sin\theta - P \cdot \Delta x \cdot \tau = \beta\ \rho\ Q(V - V) = 0$$

$$W \cdot \sin\theta = P \cdot \Delta x \cdot \tau$$

$$(A \cdot \Delta x \cdot \gamma) \cdot S = P \cdot \Delta x \cdot \tau \qquad (\theta \text{ small} \rightarrow \sin\theta \approx \tan\theta = S)$$

$$\therefore \quad \boxed{\tau = \gamma \cdot \frac{A}{P} \cdot S = \gamma\ RS}$$

式中 P =Wetted perimeter (潤周，濕周)

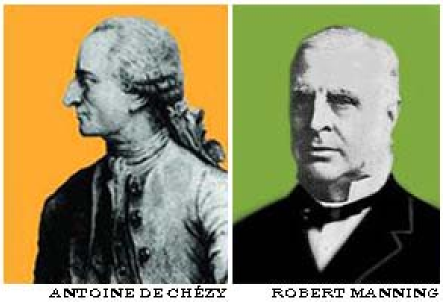
$R = \frac{A}{P}$ = Hydraulic radius (水力半徑)

● Empirical Formulas for Uniform Velocity

① Chezy eqn

$$V = C\sqrt{RS}$$

式中 C = Chezy's resistance factor



② Manning eqn

$$V = \frac{1}{n} R^{2/3} S^{1/2}$$

式中 n = Manning's roughness coefficient

● Uniform Flow Depth or Normal Depth (正常水深)

When uniform flow → Normal depth (y_n)

Normal depth(y_n)→ Obtained by Chezy or Manning eqn, but not trivial at all !!

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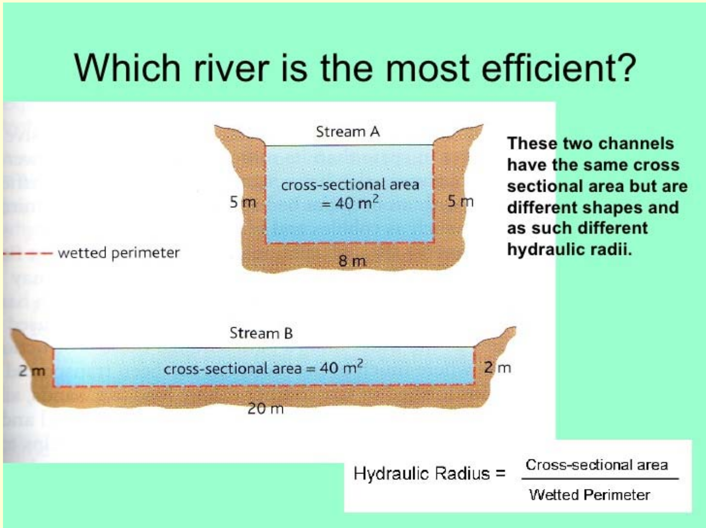
3. Best Hydraulic Cross Section (最佳水力斷面)



$$Q = AV = A\frac{1}{n}R^{2/3}S^{1/2} = \left(\frac{A^{5/3}S^{1/2}}{n}\right)\frac{1}{P^{2/3}}$$

若渠道坡度 S、糙度 n 及通水面積 A 固定，當潤周 P 為最小值時，水力半徑 $R = \frac{A}{P}$ 為最大值，此斷面輸水能力最大 (Q 最大)，故為 最佳水力斷面。

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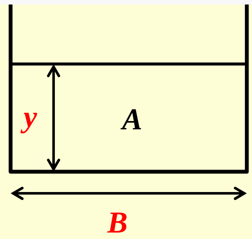
Example: 矩形斷面 之 最佳水力 設計



$$R = \frac{A}{P} = \frac{By}{B+2y} = \frac{A}{(A/y)+2y} = \frac{Ay}{(A+2y^2)}$$

When $\frac{dR}{dy} = 0 \rightarrow R_{max}$

$$\frac{d\left(\frac{Ay}{A+2y^2}\right)}{dy} = \frac{A(A-2y^2)}{(A+2y^2)^2} = 0$$

$A = 0$ (不合) 或 $A = 2y^2 = By$

$\therefore \quad B = 2y$ 時，$R_{max} = y/2$

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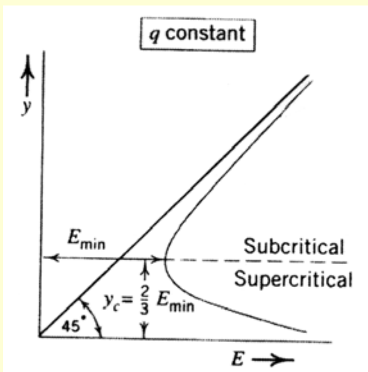
4. Specific Energy (比能)，E

$$E = y + \frac{V^2}{2g} = y + \frac{1}{2g}\left(\frac{q}{y}\right)^2$$

式中 $q = \frac{Q}{B} = \frac{B \cdot y \cdot V}{B} = y \cdot V$

(for wide rectangular channel)

- **Specific Energy Diagram (比能圖)**



Specific energy diagram

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When $\frac{dE}{dy} = 1 - \frac{q^2}{gy^3} = 0$

$\rightarrow q^2 = y^2V^2 = gy^3$

$\rightarrow V^2 = gy \quad \Rightarrow \frac{V}{\sqrt{gy}} = Fr = 1$ (Critical flow)

$\rightarrow E_{min} = y + \frac{1}{2g}\left(\frac{gy^3}{y^2}\right) = \frac{3}{2}y$

$\therefore$ Critical depth $y_c = \frac{2}{3}E_{min} = \sqrt[3]{\frac{q^2}{g}}$

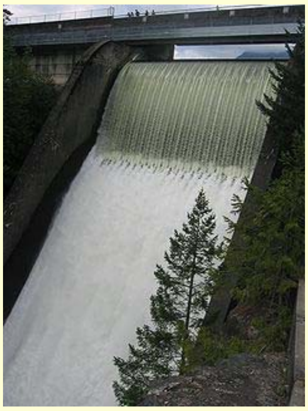
$y > y_c \Rightarrow Fr < 1$ → Subcritical flow (亞臨界流)

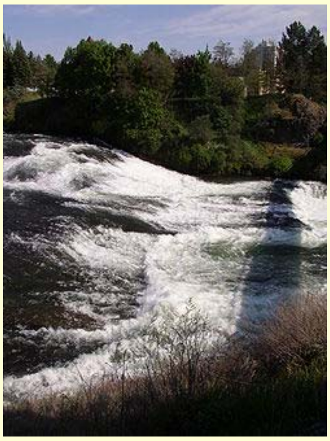
$y < y_c \Rightarrow Fr > 1$ → Supercritical flow (超臨界流)

- **Application of Specific Energy Diagram**
  - ① Flow over a hump (凸起物)
  - ② Channel contraction (窄縮)

Supercritical and Subcritical Flows (超臨界流與亞臨界流)

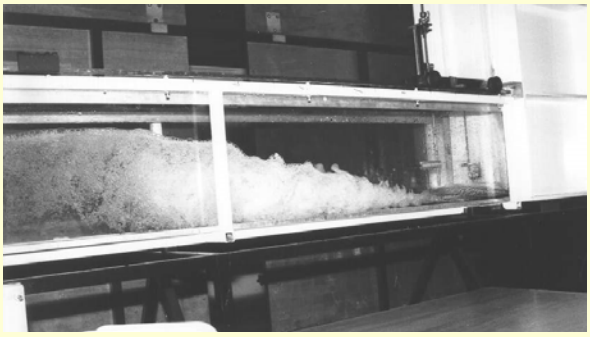
- **Supercritical flow**





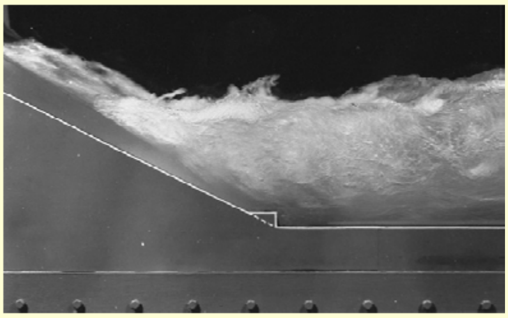
- **Subcritical flow**

● **Supercritical flow → Subcritical flow (Hydraulic Jump)**

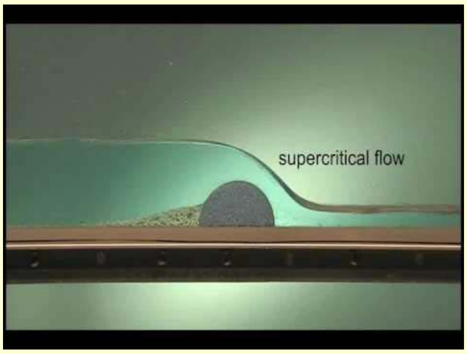


● **Subcritical flow → Supercritical flow**

● **Supercritical flow → Subcritical flow (Hydraulic Jump)**



● **Subcritical flow → Supercritical flow**



5. Hydraulic Jump (水躍)

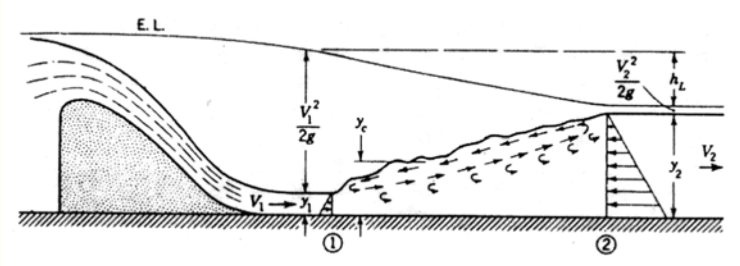
- Supercritical flow → Subcritical flow ⇨ Hydraulic Jump

Hydraulic jump below the spillway (水壩洩洪道)

Hydraulic jump in a laboratory flume (實驗水槽)

Hydraulic Jump

- Conjugate depth (共軛水深)



Momentum eqn: $F_x = F_1 - F_2 = \rho\, q(V_2 - V_1)$

in which $q = y_1V_1 = y_2V_2 \quad \Rightarrow \quad V_1 = \frac{q}{y_1}, V_2 = \frac{q}{y_2}$

$$F_1 = \frac{\gamma}{2}y_1^2, \quad F_2 = \frac{\gamma}{2}y_2^2$$

$$\Rightarrow \frac{\gamma}{2}y_1^2 - \frac{\gamma}{2}y_2^2 = \rho\, q^2\left(\frac{1}{y_2} - \frac{1}{y_1}\right)$$

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Solving this eqn yields:

$$\frac{y_2}{y_1} = \frac{1}{2}\left[-1 + \sqrt{1 + \frac{8q^2}{gy_1^3}}\right]$$

$$= \frac{1}{2}\left[-1 + \sqrt{1 + 8 \cdot \frac{V_1^2}{gy_1}}\right]$$

$$= \frac{1}{2}\left[-1 + \sqrt{1 + 8Fr_1^2}\right]$$

in which $Fr_1 = \frac{V_1}{\sqrt{gy_1}}$

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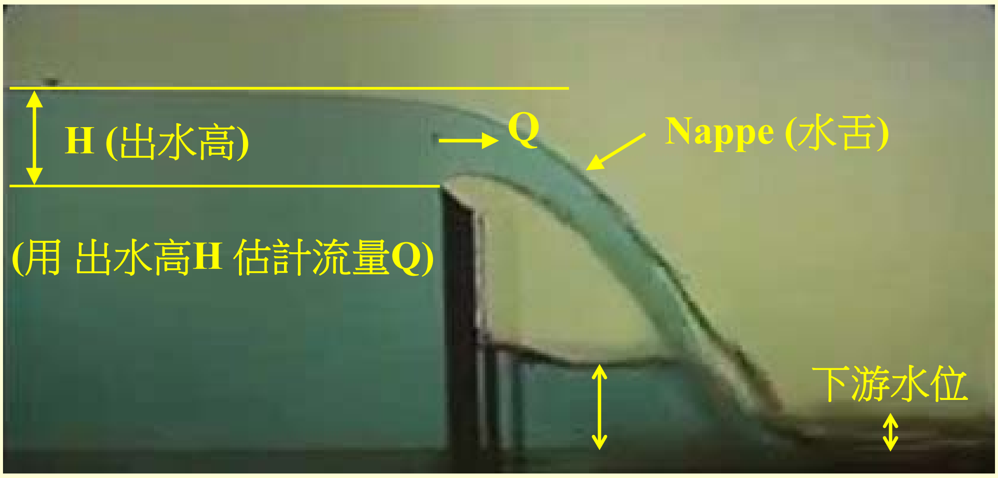
6. Measurement of Open-Channel Flow

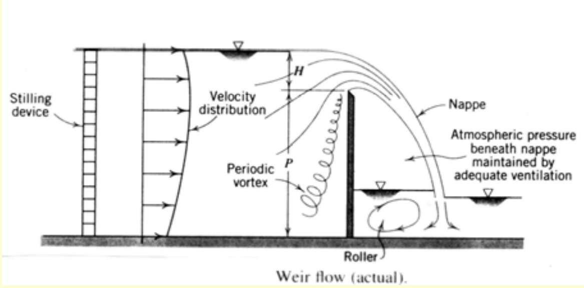
- **Sharp-crested Weir (銳緣堰)**

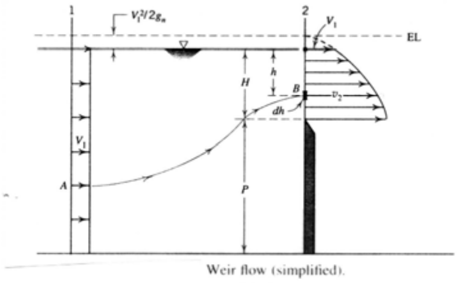
How to measure flow discharge in an open channel?

Sharp-crested weir (銳緣堰) to generate nappe flow (舌流)



① **Rectangular weir (矩形堰)**





$$H + 0 + \frac{V_1^2}{2g} = (H - h) + 0 + \frac{v_2^2}{2g} \qquad v_2 \cong \sqrt{2gh}$$

$$Q = \int v_2 dA = \int_0^H \sqrt{2gh} \cdot B \cdot dh = \frac{2}{3} \cdot \sqrt{2g} \cdot B \cdot H^{3/2}$$

$$\therefore \quad Q = C_W \cdot \frac{2}{3} \cdot \sqrt{2g} \cdot B \cdot H^{3/2}$$

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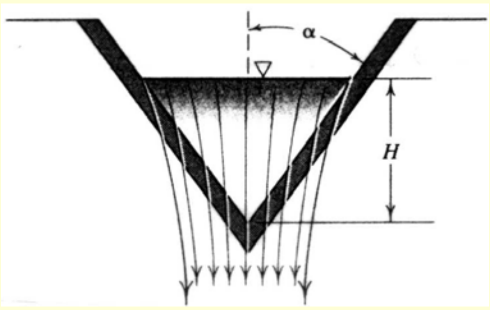
② **Triangular weir (三角堰)**

$$Q = C_W \cdot \frac{8}{15} \cdot \tan\alpha \cdot \sqrt{2g} \cdot H^{5/2}$$



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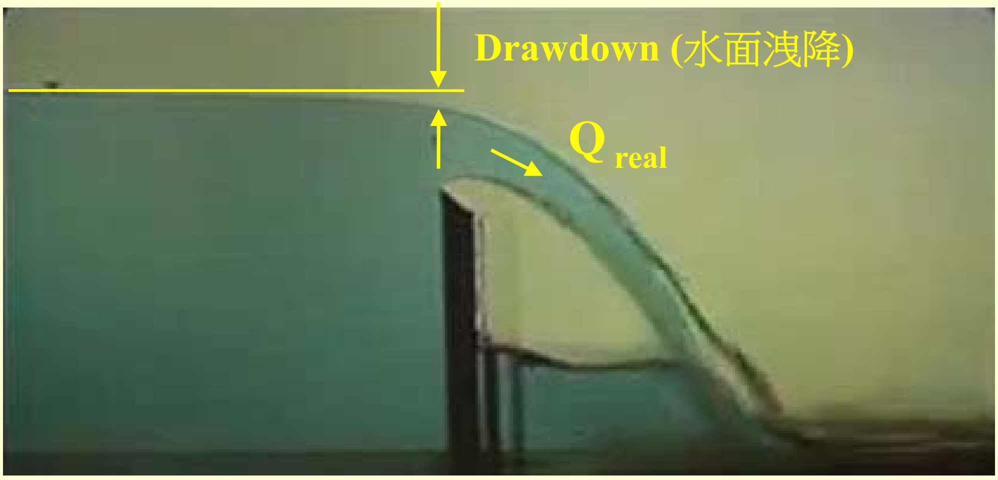
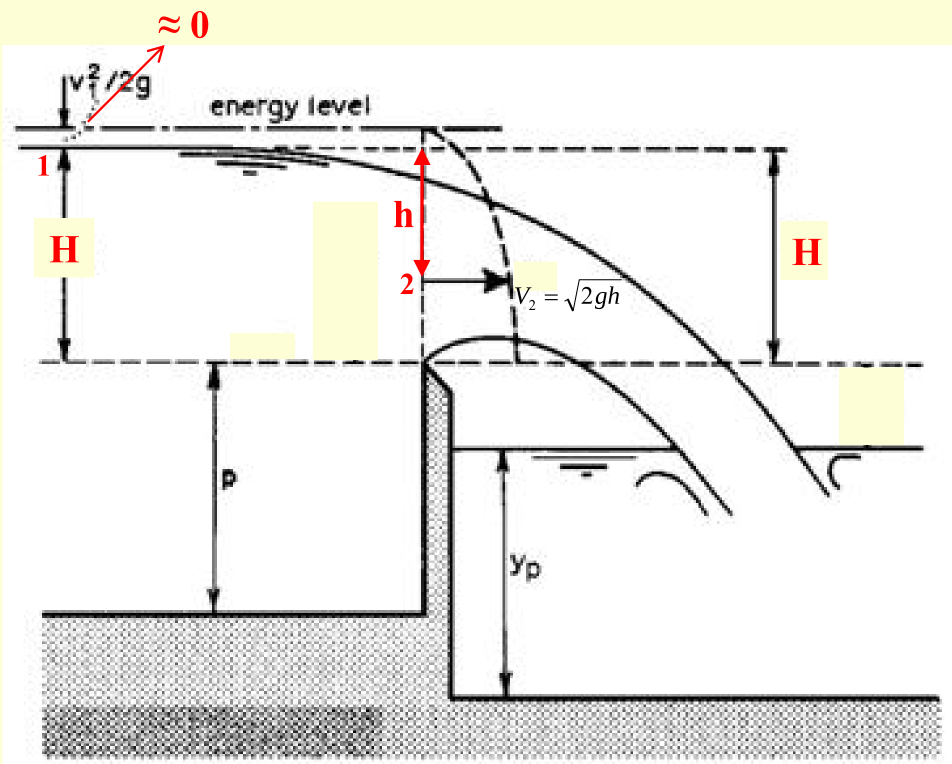
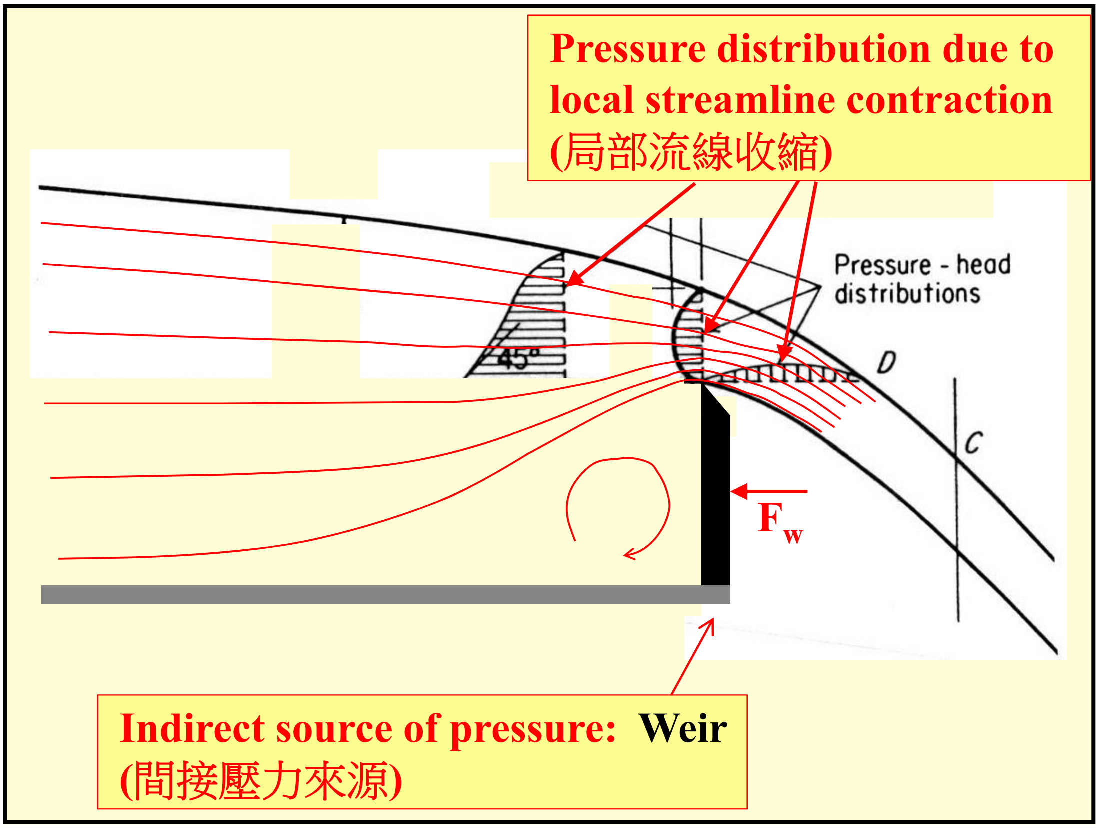
Why actual flow is smaller than idealized flow?






Pressure distribution due to local streamline contraction (局部流線收縮)
Pressure - head distributions
45°
D
C
F_w
Indirect source of pressure: Weir (間接壓力來源)