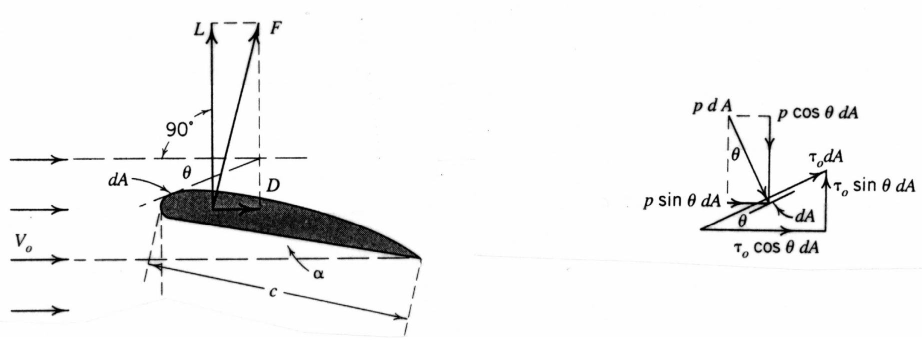
10. Drag and Lift

1. Definitions

●物體在流體中移動遭受到平行移動方向之阻力稱為拖曳力 (Drag force)

●物體在流體中移動遭受到垂直移動方向之力稱為昇力 (Lift force)



$$dF_D = PdA \cdot \sin\theta + \tau\, dA \cdot \cos\theta$$

$$dF_L = -PdA \cdot \cos\theta + \tau dA \cdot \sin\theta$$

⇒ Integration over the surface of the object:

$$F_D = \iint_A \left(P \cdot \sin\theta + \tau \cdot \cos\theta\right) dA$$

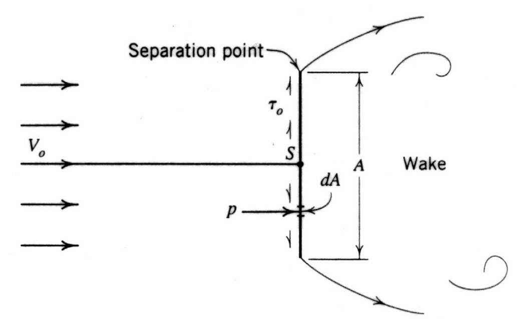
$$F_L = \iint_A \left(-P \cdot \cos\theta + \tau \cdot \sin\theta\right) dA$$

● Discussion：

① Drag： $F_D = \iint_A \left(\underbrace{P\sin\theta}_{(1)} + \underbrace{\tau\cos\theta}_{(2)} \right) dA$

(1) $P\sin\theta$ ：Pressure drag (or Form drag) → caused by form (shape) and separation

e.g. Thin plate normal to flow

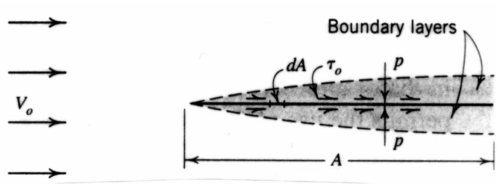


$$F_D = \iint_A PdA$$

F_D 全部由 Pressure造成

(2) $\tau\cos\theta$ ：Friction drag (or Skin drag, Viscous drag) → caused by resistance of boundary layer

e.g. Thin plate parallel to flow

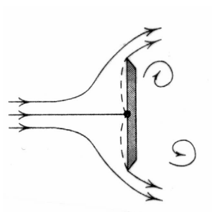


$$F_D = \iint_A \tau \, dA$$

F_D 全部由 Friction造成

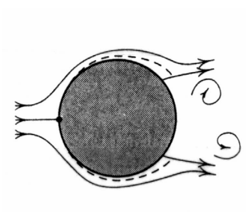
Example:

1. disk



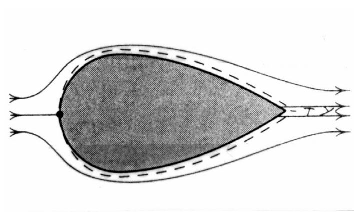
$$F_D = \iint_A P dA = F_{DP}$$

2. sphere



$$F_D \cong \frac{1}{3} F_{DP}$$

3. streamlined body



$$F_D \cong \frac{1}{40} F_{DP}$$

- Viscosity為引起Drag force之主因

  ⇒① Friction effect on surface → 引起 Friction drag

  ② Energy dissipation by surface resistance → Seperation → Low-pressure wake → 引起 Pressure drag

② Lift： $F_L = \iint_A \left(-P\cos\theta + \tau\sin\theta\right) dA$

Usually $\tau\sin\theta << P\cos\theta$ ⇒ $F_L \cong -\iint_A P\cos\theta \, dA$

2. Dimensional Analysis

$$F_D = f(A, \rho, \mu, V, E)$$

$$F_L = f(A, \rho, \mu, V, E)$$

where A= Projection area on a plane normal to flow direction

V = Moving velocity

ρ , μ , $E \equiv$ Fluid properties

⇒ Dimensional Analysis

$$F_D = C_D \cdot \frac{A\rho V^2}{2}$$

$$F_L = C_L \cdot \frac{A\rho V^2}{2}$$

where $C_D \equiv$ Drag coef. $= f(\mathrm{Re}, \mathrm{M})$

$C_L \equiv$ Lift coef. $= f(\mathrm{Re}, \mathrm{M})$

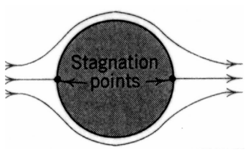
$\mathrm{Re} = \rho\sqrt{A}V / \mu$, $\mathrm{M} = \rho V^2 / E$

- For incompressible fluid → Re dominant.
- For compressible fluid → M dominant.

3. Drag

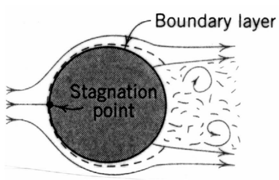
① Separation

Ideal fluid：



No resistance → No energy dissipation

Real fluid：



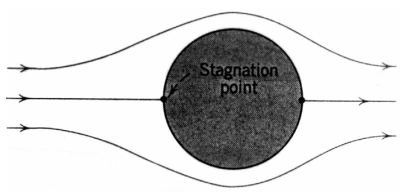
Shear resistance → Energy dissipation → Momentum reduced

→ Rest → Separation

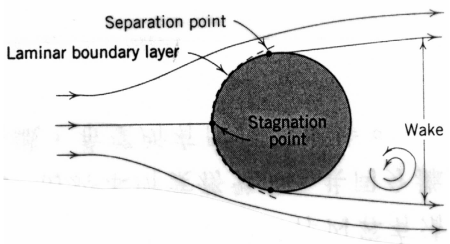
② ● Laminar B.L. → Wider wake (Momentum flux weaker)

● Turbulent B.L. → Narrower wake (Momentum flux stronger)

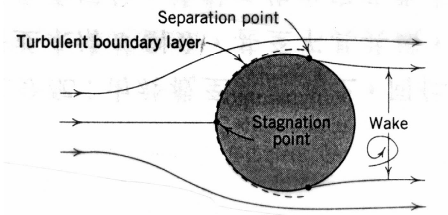
$\mathrm{Re} \le 10$



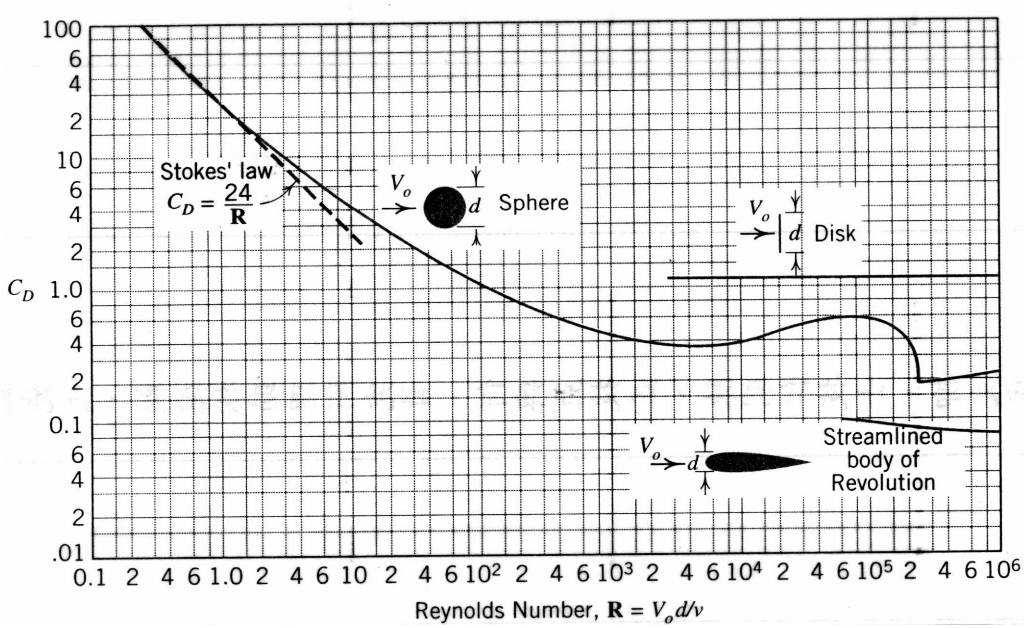
$\mathrm{Re} = 10^3 \sim 2.5\times10^5$



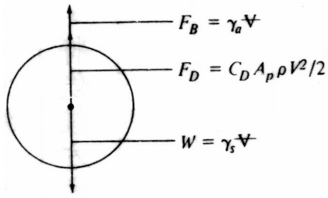
$\mathrm{Re} > 2.5\times10^5$



- C_D for sphere



Example：Terminal Velocity (終端速度)



Terminal velocity → Force Balance → $F_B + F_D = W$

$$F_D = W - F_B = (\gamma_s - \gamma)\left(\frac{4}{3}\right)\pi r^3 = \frac{1}{6}\pi D^3(\gamma_s - \gamma) \ldots\ldots\ldots\ldots\ldots\ldots \text{①}$$

$$F_D = C_D \frac{A\rho V^2}{2} \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots ②$$

$$① = ② \qquad \Rightarrow \qquad V = \left[\frac{(\gamma_s - \gamma)\left(\frac{4}{3}\right) D}{C_D \cdot \rho} \right]^{1/2}$$

Case 1：for $\mathrm{Re} < 10 \quad \Rightarrow \quad C_D = 24/\mathrm{Re}$

$$V = \left[\frac{(\gamma_s - \gamma)\left(\frac{4}{3}\right) D \cdot \frac{V \cdot D}{\nu}}{24 \cdot \rho \cdot V} \right] = \frac{1}{18} \cdot \frac{(\gamma_s - \gamma)}{\rho} \cdot \frac{D^2}{\nu}$$

Case 2：Trial-and-error (試誤法)

A 50-mm sphere (S.G.=1.3) dropping in water

$$V = \left[\frac{(0.3) g \left(\frac{4}{3}\right)(0.05)}{C_D} \right]^{1/2} = \frac{0.44}{\sqrt{C_D}} \qquad m/s$$

(1) Guess $C_D = 1.0$

$$V = \frac{0.44}{\sqrt{1}} = 0.44 \quad m/s$$

$$\rightarrow \quad \mathrm{Re} = \frac{(0.44 m/s)(0.05 m)(10^3\, kg/m^3)}{1 \times 10^{-3} \quad N \cdot s/m^2} = 2.2 \times 10^4$$

(2) 查 Fig. 11.9: $\mathrm{Re} = 2.2 \times 10^4 \rightarrow C_D = 0.5$

$$V = \frac{0.44}{\sqrt{0.5}} = 0.62 \quad m/s \quad \rightarrow \quad \mathrm{Re} = 3.1 \times 10^4$$

(3) 查 Fig. 11.9: $\mathrm{Re} = 3.1 \times 10^4 \rightarrow C_D = 0.52$ (OK)

(4) $V = \frac{0.44}{\sqrt{0.52}} = 0.61 \quad m/s$

4. Lift

- Neglecting viscosity → Ideal fluid → Potential flow

  Stream function ψ → Useful tool

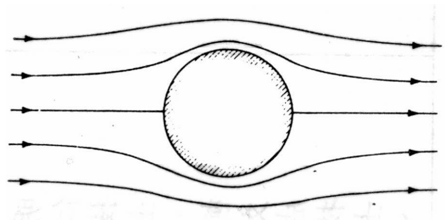
<u>Example</u>:

Linear Superposition of ① Uniform flow

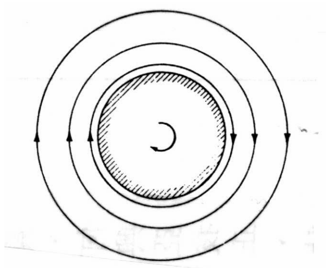
② Doublet (Source + Sink)

③ Free Vortex

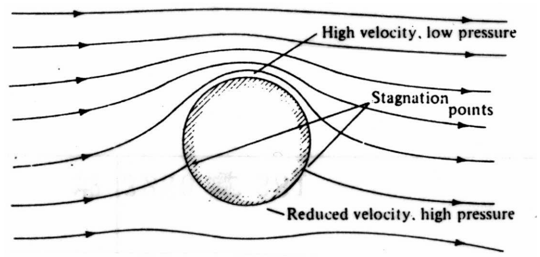
① + ②



+ ③



⇒



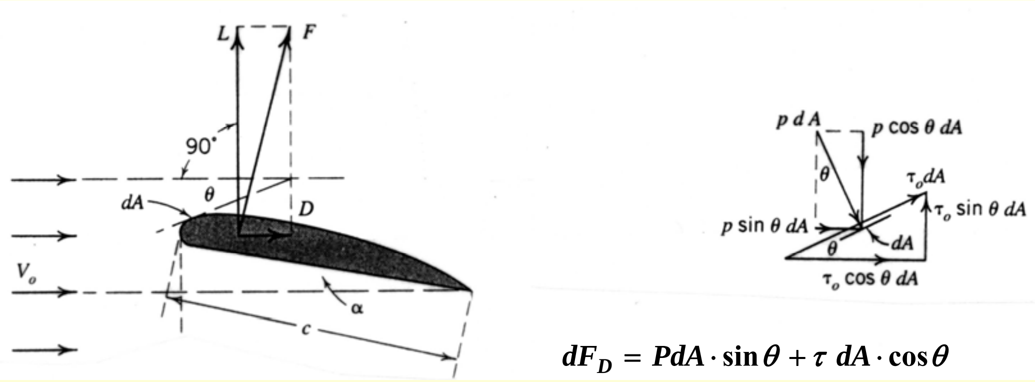
$$\psi = \underbrace{Ur\sin\theta}_{(1)} - \underbrace{\frac{aq}{\pi}\frac{\sin\theta}{r}}_{(2)} + \underbrace{\frac{\Gamma}{2\pi}\ln r}_{(3)}$$

可計算得 $\Rightarrow \begin{cases} F_D = 0 & \text{(左右對稱)} \\ F_L = \rho U \Gamma & \text{(向上)} \end{cases}$

10. Drag and Lift

1. Definitions

- 物體在流體中移動遭受到平行移動方向之阻力稱為 拖曳力 (Drag force)
- 物體在流體中移動遭受到垂直移動方向之力稱為 昇力 (Lift force)



$$dF_D = PdA \cdot \sin\theta + \tau\ dA \cdot \cos\theta$$

$$dF_L = -PdA \cdot \cos\theta + \tau dA \cdot \sin\theta$$

⇨ Integration over the surface of the object:

$$F_D = \iint_A (P \cdot \sin\theta + \tau \cdot \cos\theta) dA$$

$$F_L = \iint_A (-P \cdot \cos\theta + \tau \cdot \sin\theta) dA$$

- Discussion of drag and lift forces

① Drag： $F_D = \iint_A \left(\underbrace{P\sin\theta}_{(1)} + \underbrace{\tau\cos\theta}_{(2)} \right) dA$

(1) $P\sin\theta$： Pressure drag (or Form drag ，形狀阻力)

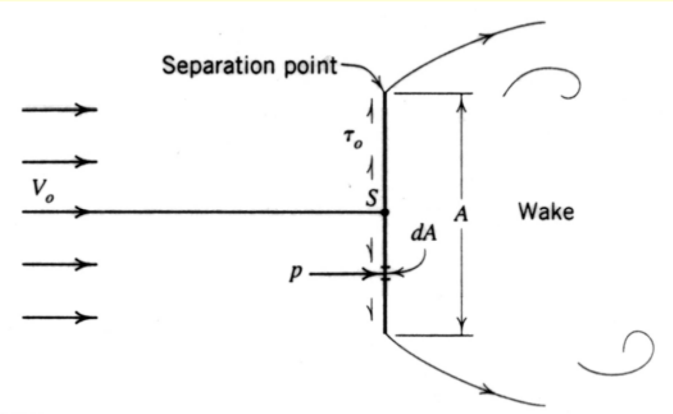
→ caused by form (shape) and separation (分離流)

(2) $\tau\cos\theta$ ： Friction drag (or Skin drag 表面阻力, Viscous drag)

→ caused by resistance of boundary layer

(1) $P\sin\theta$: **Pressure drag (or Form drag ，形狀阻力)**

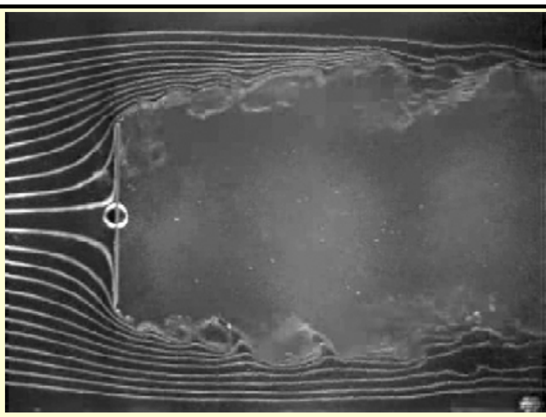
An extreme example: Thin plate normal to flow (see photos on next page)

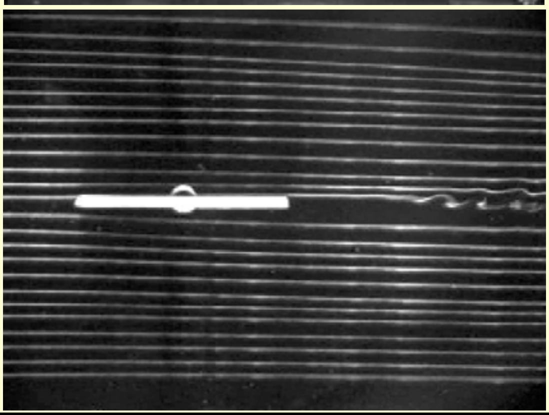


$$F_D = \iint_A P dA$$

F_D 全部由 **Pressure** 所造成

3

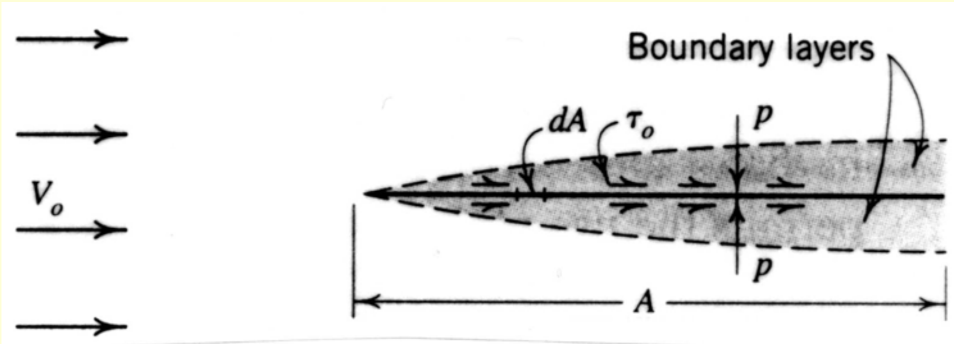




4

(2) $\tau\cos\theta$ ：Friction drag (or Skin drag 表面阻力, Viscous drag)

An extreme example: Thin plate parallel to flow



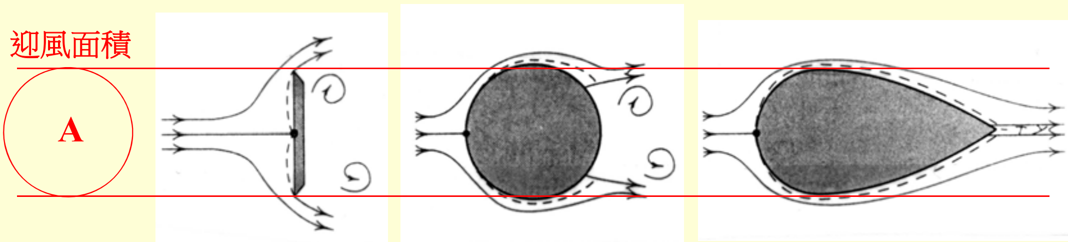
$$F_D = \iint_A \tau \, dA$$

F_D 全部由 Friction 所造成

Example:

1. disk　　2. sphere　　3. streamlined body

迎風面積



$$F_D = \iint_A P dA = F_{DP} \qquad F_D \cong \frac{1}{3} F_{DP} \qquad F_D \cong \frac{1}{40} F_{DP}$$

- **Drag force 與 Viscosity 息息相關**

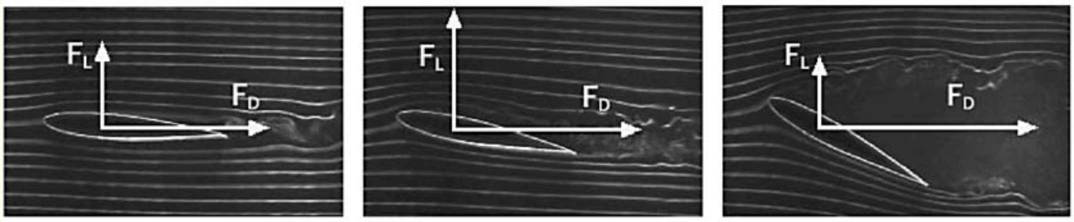
  ➩ ① Viscous shear stress on surface → 引起 Skin (or friction) drag

  ② Viscous shear stress = 0 at the surface → Separation (分離流)

  → Low-pressure wake (尾跡) → 引起 Form (or pressure) drag

Flow separation and the wake region during flow over a tennis ball.

At large angles of attack (usually larger than 15°), flow may separate completely from the top surface of an airfoil, reducing lift drastically and causing the airfoil to stall.



(a) 5° (b) 15° (c) 30°

Discussion of Lift force:

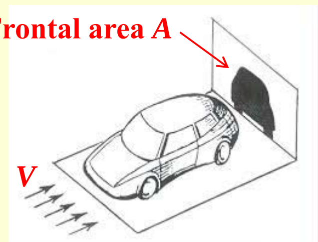
② **Lift**： $F_L = \iint_A (-P\cos\theta + \tau\sin\theta)dA$

Usually: $\tau\sin\theta << P\cos\theta \Rightarrow F_L \cong -\iint_A P\cos\theta dA$

2. Dimensional Analyses of Drag and Lift Forces (因次分析)

$$F_D = f(A, \rho, \mu, V, E)$$

$$F_L = f(A, \rho, \mu, V, E)$$



where A = Frontal area (迎風面積), which is **projection area** on a plane normal to flow direction

V = Approaching (or moving) velocity 接近(或移動)速度

ρ, μ, E ≡ Fluid properties

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➪ **Dimensional Analysis** (n – k = 3 ➪ π_1, π_2, π_3)

$$F_D = C_D \cdot \frac{A\rho V^2}{2}$$

$$F_L = C_L \cdot \frac{A\rho V^2}{2}$$

where $C_D \equiv$ Drag coef. $= f(\mathrm{Re}, \mathrm{M})$

$C_L \equiv$ Lift coef. $= f(\mathrm{Re}, \mathrm{M})$

$\mathrm{Re} = \rho\sqrt{A}V/\mu$, $\mathrm{M} = \rho V^2/E$

- **For incompressible fluid → Re dominant.**
- **For compressible fluid → M dominant.**

9

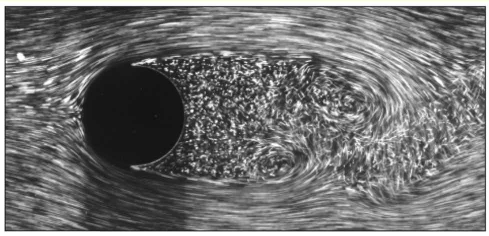
FLOW OVER CYLINDERS AND SPHERES

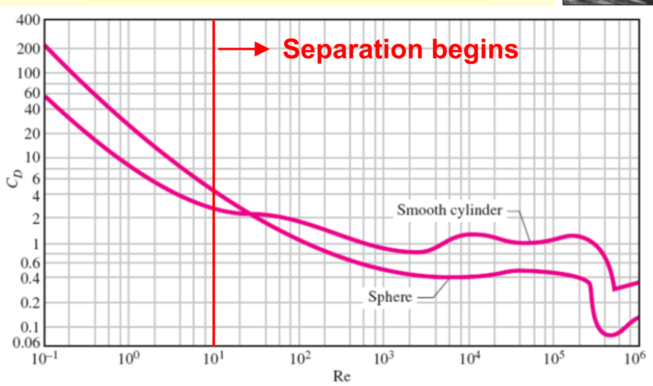
Flow over cylinders and spheres is frequently encountered in practice.

Many sports such as soccer, tennis, and golf involve flow over spherical balls.

At **very low velocities**, the fluid completely wraps around the cylinder.

Flow in the **wake region (尾跡區)** is characterized by **periodic vortex formation and low pressures**.



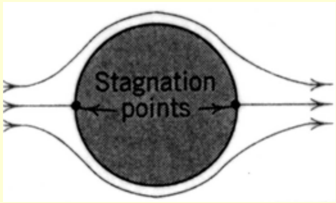


Laminar boundary layer separation with a turbulent wake; flow over a circular cylinder.

3. Determination of Drag Coefficient C_D

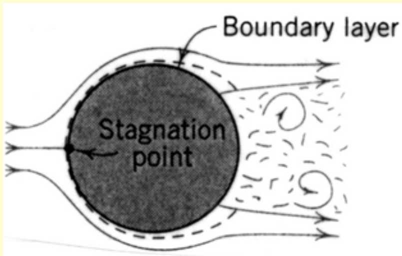
① Mechanism of Separation (分離流之機制)

Ideal fluid



No viscosity → No separation

Real fluid



Adverse pressure gradient → Boundary viscous shear stress = 0
→ Separation!!

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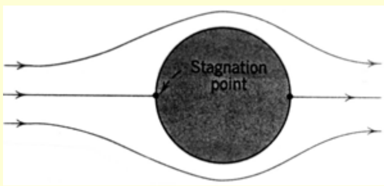
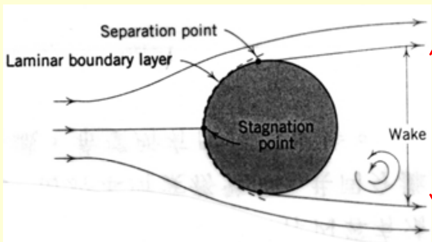
② ● **Laminar B.L.** → **Wider wake (Momentum flux weaker)**

● **Turbulent B.L.** → **Narrower wake (Momentum flux stronger)**

$Re \le 10$
(No separation)

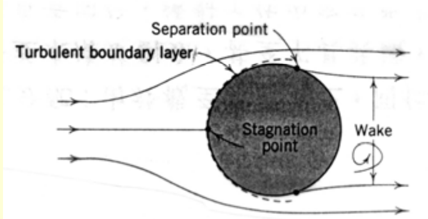


$Re = 10^3 \sim 2.5\times10^5$



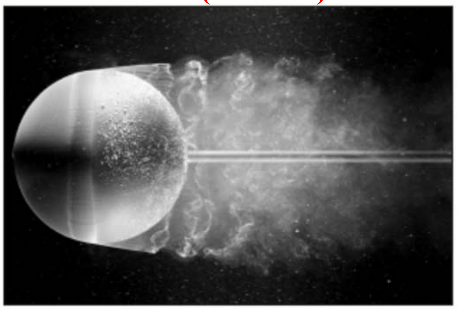
Wider Wake

$Re > 2.5\times10^5$



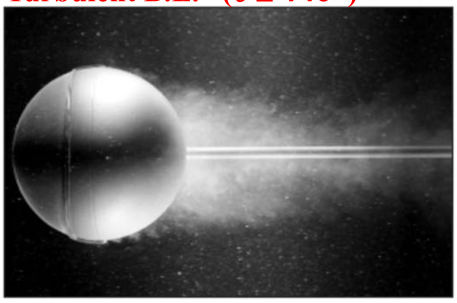
Narrower Wake

Laminar B.L. ($\theta \cong 80^\circ$)



(a)

Turbulent B.L. ($\theta \cong 140^\circ$)



(b)

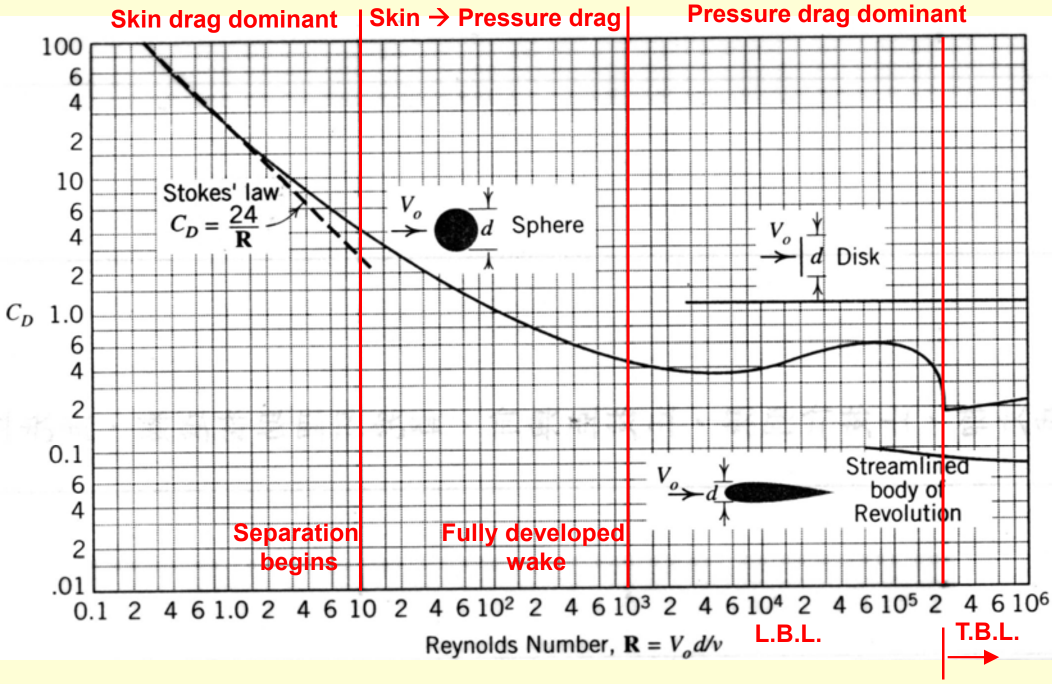
Flow separation occurs at about $\theta \cong 80^\circ$ (measured from the front stagnation point of a cylinder) when the boundary layer is *laminar* and at about $\theta \cong 140^\circ$ when it is *turbulent*.

The delay of separation in turbulent flow is caused by the rapid fluctuations of the fluid in the transverse direction, which enables the turbulent boundary layer to travel farther along the surface before separation occurs, resulting in a narrower wake and a smaller pressure drag.

Flow visualization of flow over
(a) Laminar boundary layer;
(b) Turbulent boundary layer.

The delay of boundary layer separation is clearly seen by comparing these two photographs.

- **C_D for sphere** $\left(F_D = C_D \cdot \dfrac{A\rho V^2}{2}\right)$



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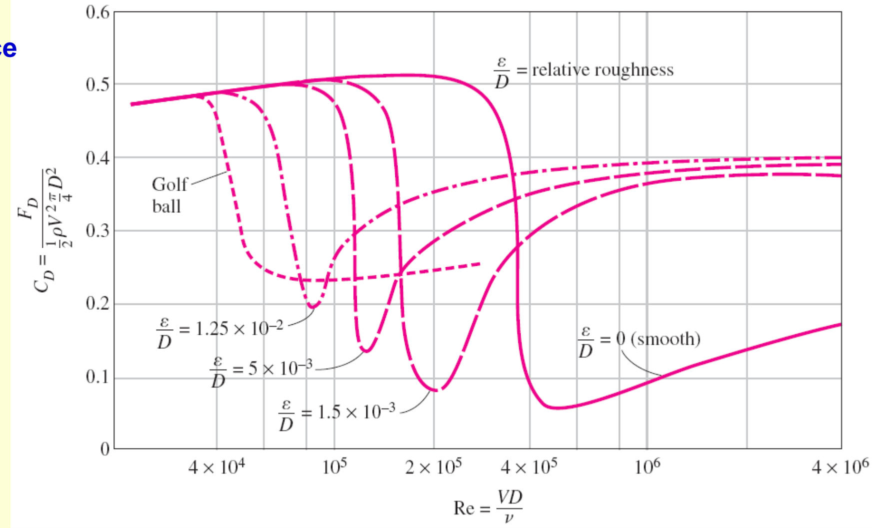
Effect of Surface Roughness (表面糙度)

Surface roughness, in general, increases the drag coefficient in turbulent flow.

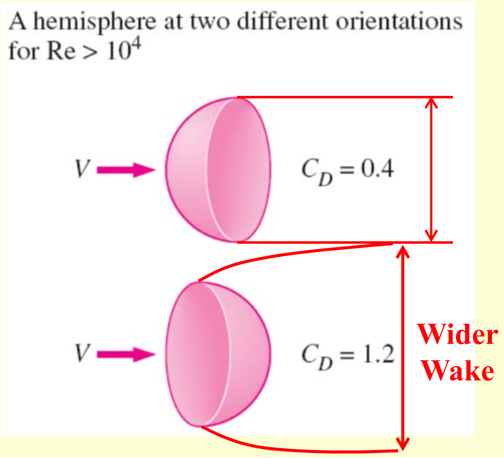
This is especially the case for streamlined bodies.

For blunt bodies such as a circular cylinder or sphere, however, an increase in the surface roughness may *increase* or *decrease* the drag coefficient depending on Reynolds number.

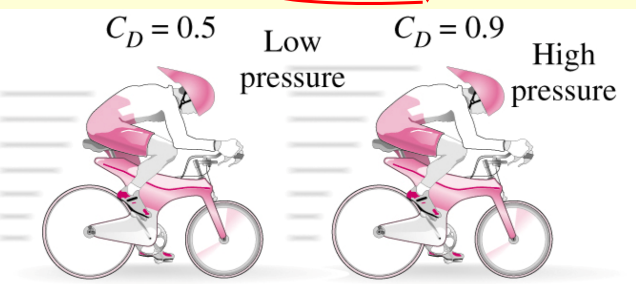
Effect of surface roughness on C_D of a sphere.



Effects of Orientation and Position (方向與位置)



The drag coefficient of a body may change drastically by changing the body's orientation (and thus shape) relative to the direction of flow.



The drag coefficients of bodies following other moving bodies closely can be reduced considerably due to drafting (i.e., entering into the low pressure region created by the body in front).

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2018冬季奧運 日本女子競速滑冰隊 高木姐妹 (3金、1銀、1銅)

https://www.nicovideo.jp/watch/sm32795076

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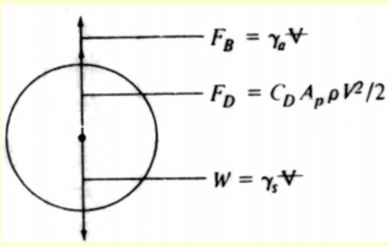
Example: Determination of Terminal Fall Velocity (終端沉降速度)



Terminal velocity → Force Balance → $W = F_B + F_D$

$$F_D = W - F_B = (\gamma_s - \gamma)\left(\frac{4}{3}\right)\pi r^3 = \frac{1}{6}\pi D^3(\gamma_s - \gamma) \qquad ①$$

$$F_D = C_D \frac{A\rho V^2}{2} \qquad ②$$

$$① = ② \quad ⇨ \quad V = \left[\frac{(\gamma_s - \gamma)\left(\frac{4}{3}\right)D}{C_D \cdot \rho}\right]^{1/2}$$

18

Case 1 : for $\mathrm{Re} < 10 \Rightarrow C_D = 24/\mathrm{Re}$ (Stokes' law)

$$V = \left[\frac{(\gamma_s - \gamma)\left(\frac{4}{3}\right)D \cdot \frac{V \cdot D}{\nu}}{24 \cdot \rho \cdot V}\right] = \frac{1}{18} \cdot \frac{(\gamma_s - \gamma)}{\rho} \cdot \frac{D^2}{\nu}$$

Case 2 : for Re > 10, use trial-and-error method (試誤法)

Given: A 50 mm sphere (with S.G.=1.3) dropping in water

$$V = \left[\frac{(0.3)g\left(\frac{4}{3}\right)(0.05)}{C_D}\right]^{1/2} = \frac{0.44}{\sqrt{C_D}} \quad m/s$$

(1) Initial guess: $C_D = 1.0$

$$V = \frac{0.44}{\sqrt{1}} = 0.44 \quad m/s$$

$$\rightarrow \mathrm{Re} = \frac{(0.44m/s)(0.05m)(10^3 kg/m^3)}{1 \times 10^{-3} \quad N \cdot s/m^2} = 2.2 \times 10^4$$

(2) 查 圖: $\mathrm{Re} = 2.2 \times 10^4 \rightarrow C_D = 0.5$

$$V = \frac{0.44}{\sqrt{0.5}} = 0.62 \quad m/s \quad \rightarrow \quad \mathrm{Re} = 3.1 \times 10^4$$

(3) 查圖: $\mathrm{Re} = 3.1 \times 10^4 \rightarrow C_D = 0.52$ (OK)

(4) $V = \dfrac{0.44}{\sqrt{0.52}} = \boxed{0.61 \quad m/s}$

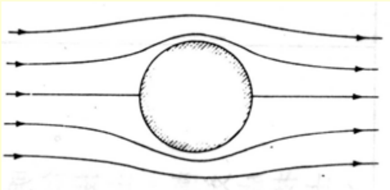
4. Determination of Lift

- **Pressure lift dominant → Viscous lift negligible → Ideal fluid → Potential flow**

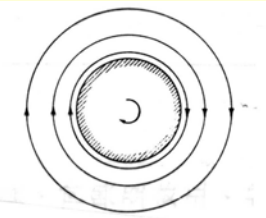
  Stream function ψ → Useful tool for analyzing lift (but not drag!!)

Example: Linear Superposition of ① Uniform flow
② Doublet (Source + Sink)
③ Free Vortex

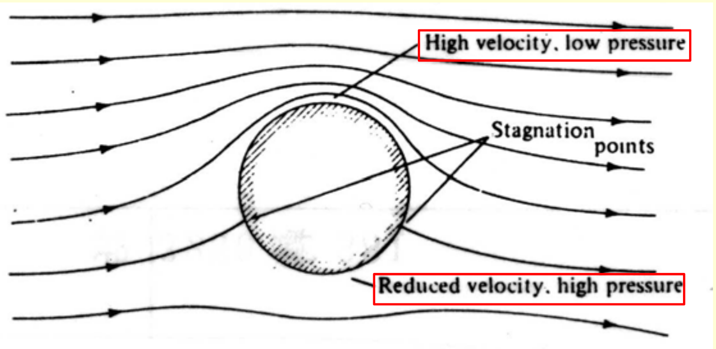
① + ②



\+ ③



⇒



$$\psi = \underbrace{U r \sin\theta}_{(1)} - \underbrace{\frac{aq}{\pi}\frac{\sin\theta}{r}}_{(2)} + \underbrace{\frac{\Gamma}{2\pi}\ln r}_{(3)}$$

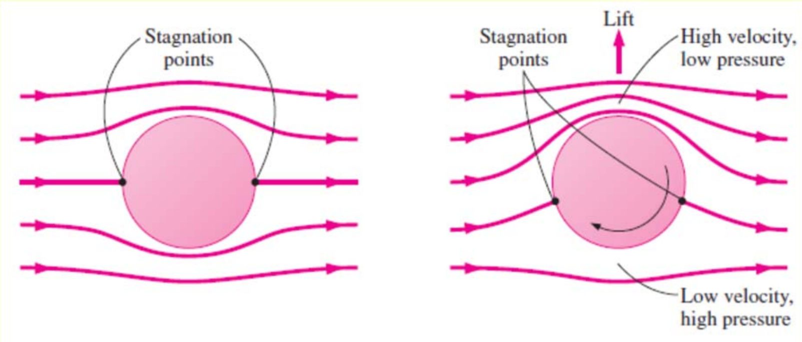
積分周邊面積之壓力，可計算 ： ⇒ $F_L = \rho U \Gamma$ (向上)

Lift Generated by Spinning

Magnus effect (馬格納斯效應): The phenomenon of producing lift by the rotation of a solid body.

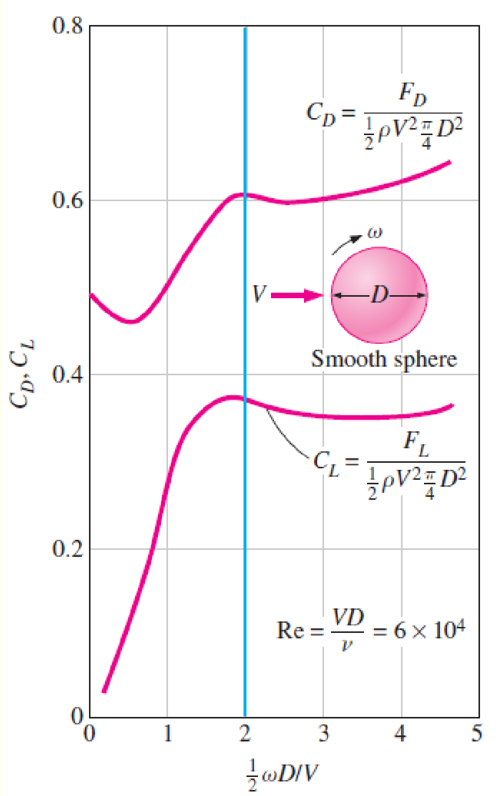
When the ball is not spinning, the lift is zero because of top–bottom symmetry. But when the cylinder is rotated about its axis, the cylinder drags some fluid around because of the no-slip condition and the flow field reflects the superposition of the spinning and nonspinning flows.

Magnus (German, 1802 – 1870)



(*a*) Potential flow over a stationary cylinder (*b*) Potential flow over a rotating cylinder

Generation of lift on a rotating circular cylinder for the case of "idealized" potential flow (the actual flow involves flow separation in the wake region).



Note that C_L strongly depends on the rate of rotation, especially at low angular velocities.

The effect of rate of rotation on C_D is small.

The variation of lift and drag coefficients of a smooth sphere with the nondimensional rate of rotation for $\text{Re} = VD/\nu = 6 \times 10^4$.