流 體 力 學 講 義

1. Fundamentals

1. Introduction

(1)流體力學已變成多種領域之基礎學科

①**傳統工程** (Conventional Engineering)

②**海洋學** (Oceanography)

③**航太科學**(**火箭設計**) (Astronautics)

④**氣象學**(**大氣流動**) (Meteorology)

⑤**太陽物理** (Solar physics)

⑥**醫學**(**血液流動**) (Medicine)

(2)歷史概述

①**最早之流體與生活之關係**(**飲水、灌溉、航運、水利**)

(**方式：井、渠道、水車、抽水、水管**)

②Archimedes (287-212 B.C.) **阿基米德→發現浮力**

③Leonardo da Vinci (1425-1519) **達文西**

最早設計建造 canal lock (near Milan) **閘門**

④Galileo, Pascal, Newton, Pitot, Bernoulli, Euler, d'Alembert

累積知識 理論←實驗

理論與實驗之矛盾與衝突→(1) Hydrodynamics **流體動力學**(**偏重數學**)

(2) Hydraulics **水力學**(**偏重實際應用**)

⑤Navier-Stoke (19**世紀中葉**)

ideal fluid motion → real fluid motion (viscous fluid) **考慮流體之黏滯性**

⑥**同一時期很多**Hydraulic Research → Empirical formulas or Tabular form

Physical facts ✕Empicical formula

⑦19**世紀末，航空工業→空氣動力學**

流力長足進步，由於(1)Reynolds **理論實驗**

(2)Rayleigh **因次分析**

(3)Froude, Reynolds.... **模型試驗**

(4)**航空學** (aeronautics) **理論實驗**

長足進步 Kutta, Prandtl**等**

⑧**流體力學上最重要之貢獻：**Prandtl (1904) → Boundary Layer (**邊界層**)

⑨Application of fundamental laws of <i>Mechanics (**力學**)

<ii>Thermodynamics (**熱力學**)

<iii>Experimentation and Simulation (**實驗**)

<iv>Fluid properties (**流體基本性質**)

成為⇨Modern Fluid Mechanics

⑩20**世紀：流體力學是** <i>**一種科學**

<ii>**一種工程**

以合理(Rational)**方法尋求答案**

電腦發展→使流體力學更進步

2. Fluid Properties

(1) Fluid (流體)包括 <i>Liquid (**液體**)

<ii>Gas (**氣體**)

①**分子間距**(Spacing of molecules)

氣體＞液體＞固體

②**分子間內聚力**(Intermolecular cohesive forces)

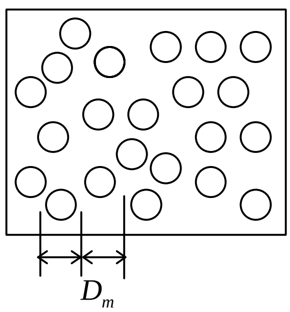
固體＞液體＞氣體

③Fluid is a <u>continuum</u> (**聯體、連續性物質**)

⇨No voids or holes in continuum. (**聯體中沒有孔隙**)

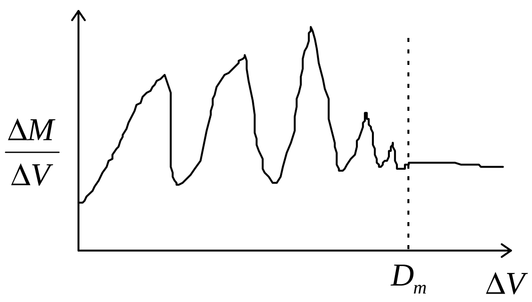
Note: Continuum assumption is valid only when:

physical length scale＞＞ average molecule spacing



i.e. $$r = \lim_{\Delta V \to V_*} \frac{\Delta M}{\Delta V}$$

$$V_* >> D_m \quad \text{(Molecule spacing)}$$



※亦即不視流體之分子結構而以流體整體之運動為討論對象

④**物質在應力作用下之反應：**(Actions under various stress)

<i>**固體承受壓力**(compression)**、張力**(tension)**或剪力**(shear)**，先產生彈性變形**(elastic deformation)**，若外力過大，便產生永久形變**(permanent distortion)**。**

<ii>**流體承受壓力時會產生彈性變形。流體幾乎不能承受任何張力，而且在剪力作用下必產生永久形變。**

⑤**流體**(Fluid)**之定義：係一種「不論多小之剪力作用下皆會產生連續永久變形」之物質。**

⑥**剪力**(shear)**→帶動流體流動**(flow)**或變形**

※流體靜止(at rest)**時，剪應力不存在→故只有壓力**(pressure)

⑦**液體**(liquid)**通常視為不可壓縮流體**(incompressible fluid)

例外：Water hammer (**水錘**) in pipe system.

(2) Fluid Properties

①Density (**密度**)， r

$$r = \lim_{\Delta V \to 0^+} \frac{\Delta M}{\Delta V}$$

②Specific Volume (**比體積**)， a

$$a = \frac{\Delta V}{\Delta M} = \frac{1}{r}$$

③Specific Gravity (**比重**)， $s.g.$

$$s.g. = \frac{r}{r_{water}}$$ for specified temperature and pressure(**定溫定壓**)

④Specific Weight(**比重量**)， g

$$g = rg$$

⑤Compressibility (**可壓縮性**)

If $Temp. = const.$ $P\uparrow, V\downarrow$ **，** $P\downarrow, V\uparrow$

⇨**實驗參見** p.12 Fig. 1-3

$$dP = -Ev\left(\frac{dV}{V}\right)$$

Ev:bulk modulus of elasticity (**彈性模數**)

例如：

$$Ev_{water} = 2.2\times10^{9}\, N/m^{2}$$

$1\times10^{6}\, N/m^{2}$ (10 atm)→ **體積僅改變**0.04% (**水之可壓縮性很小，一般可忽略**)

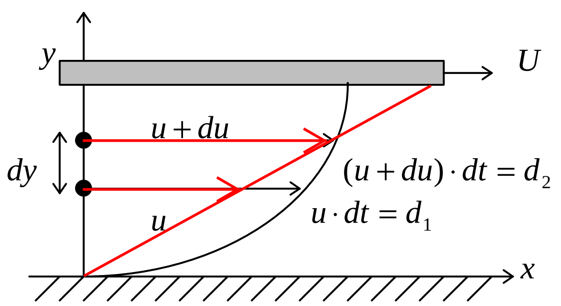
For fix amount of fluid: $M = rV$ → $dM = r\,dV + Vdr = 0$

$$\frac{dr}{r} = -\frac{dV}{V} \;\rightarrow\; dP = -Ev\left(\frac{dV}{V}\right) = Ev\left(\frac{dr}{r}\right)$$

※⑥Viscosity (**黏滯性**) **重要！**

<u>流體黏滯性之成因：</u>

流體分子間碰撞(molecular momentum exchange)**而且分子間具有內聚吸引力**(cohesion)**之故。**



No-Slip condition(不滑動條件)

①Strain (**應變**) $= \dfrac{d_2 - d_1}{dy} = \dfrac{du\cdot dt}{dy}$

②shear stress $t_{yx} \propto$ Rate of Strain

Velocity gradient＝Rate of Strain $= \dfrac{\text{Strain}}{dt} = \dfrac{(du\cdot dt/dy)}{dt} = \dfrac{du}{dy}$

$$\rightarrow t_{yx} \propto \frac{du}{dy} \qquad \therefore t_{yx} = m\frac{du}{dy}$$

此稱之為 Newton's Viscosity Law

符合牛頓黏滯律之流體稱為牛頓性流體(Newtonian fluid)

$m =$ coefficient of viscosity(**黏滯性係數**)

=dynamic viscosity (or absolute viscosity)

(**單位：英制**$= \ell b \cdot s / ft^2$ **，** **公制**$= Pa \cdot s$)

Note: ① t and m indep. of P (**與壓力之關係不顯著**)

② t causes: Velocity Gradient ($\frac{du}{dy}$)

$\Rightarrow$ Flow (Relative motion between layers)

③If fluid at rest → $u = 0$ → $\frac{du}{dy} = 0$ → $t = 0$

→No shear stress!!

④Velocity profile cannot be tangent to a solid boundary.

$\frac{du}{dy} \to \infty$ → $t \to \infty$ (impossible)

•Kinematic viscosity (**運動黏滯性**)**，** n

$$n = \frac{m}{r}$$

◎溫度對m**與**n **之影響：**

①**液體**viscosity→**主要由於分子間內聚力**(cohesion)

氣體viscosity→**主要由於分子間碰撞**(momentum exchange)

②**溫度上升→液體分子間內聚力↓**⇨m↓

溫度上升→氣體分子間碰撞↑⇨m↑

⑦Surface tension (**表面張力**)**，**Capillarity (**毛細現象**)

(1)**成因：**①Intermolecular Cohesion (**分子間內聚力**)

②Intermolecular Adhesive Force (**分子間附著力**)

(2) Negligible in many engineering problems.

→Predominant in ①capillary rise of liquid in narrow space

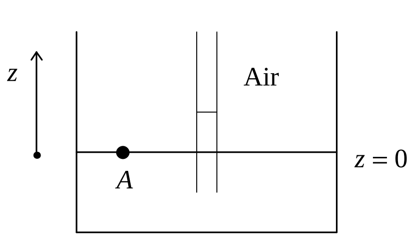
②bubble formation, liquid drop.

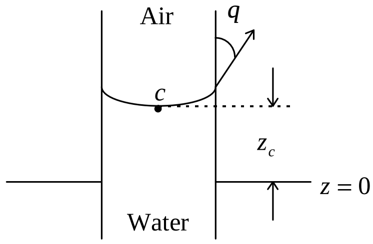
③breakup of liquid jets.

④small scale model

→**工程上：地下水**(**土壤顆粒間之水分、污染物**)

(3)**推導：(圓形毛細管)**





$$h = \frac{P}{rg} + z$$

h =piezometric head (or hydraulic head)

Static: A: $h_A = 0 + 0 = 0$

C: $h_C = \frac{P_c}{rg} + Z_c = 0$ (no flow)

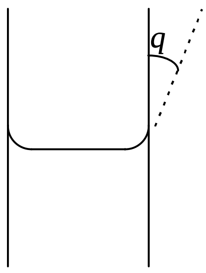
$\therefore \quad \frac{P_c}{rg} = -Z_c$ ※pressure in capillary is negative!!

s_{aw} =Tension force per unit length

$\sum F_Z = 0$ ： $(p\,r^2)\cdot P_c + s_{aw}\cdot(2p\,r)\cos q = 0$

$(p\,r^2)\cdot(-z_c\cdot r\,g) + s_{aw}\cdot(2p\,r)\cos q = 0$

$$z_c = \frac{2s_{aw}\cdot\cos q}{r\,g\cdot r}$$



通常 $q \to 0°$

$\cos q \to 1 \qquad \therefore z_c \cong \frac{2s_{aw}}{rg\cdot r}$

⑧Vapor Pressure (**蒸汽壓**) [**液體表面分子掙脫，分子撞擊形成壓力**]

Temp.↑，Vapor pressure (P_v)↑

◎Cavitation(**穴蝕**)：

(Boiling：**當**external absolute pressure imposed on liquid is equal or less than the vapor pressure of the liquid → formation of vapor bubbles)

When Velocity↑ → external pressure↓ → boiling bubbles→**形成**cavitation

⇒**氣泡高速撞擊渦輪機葉面或磨蝕管壁，造成損毀！**

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① 傳統工程 (Conventional Engineering)

② 海洋學 (Oceanography)

③ 航太科學(飛機、火箭設計) (Astronautics)

④ 氣象學(大氣流動) (Meteorology)

⑤ 醫學(血液流動) (Medicine)

⑥ 生態工程、河川棲地復育 (Ecological Engineering)

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Industrial applications
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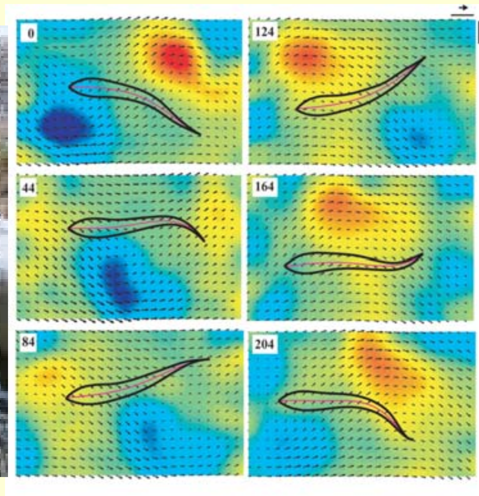
Ecological Engineering 生態工程 –
Fishway Design (魚道設計)



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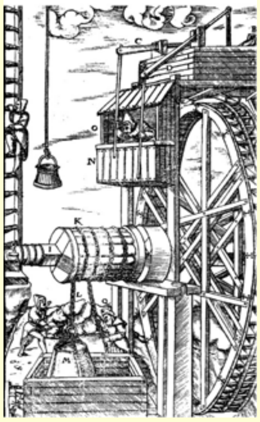
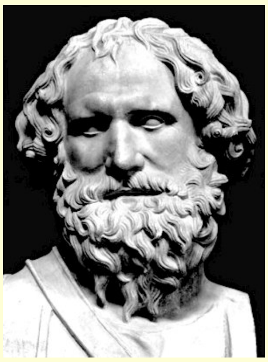
(2) 歷史概述

① 最早之流體與生活之關係(飲水、灌溉、航運、水利)
(方式：井、渠道、水車、抽水、水管)



② **Archimedes (287-212 B.C.)** 阿基米德 → 發現浮力應用



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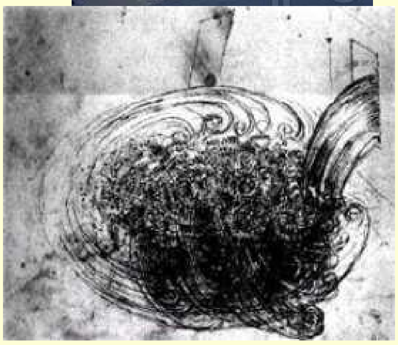
(2) 歷史概述

③ **Leonardo da Vinci (1425-1519)** 達文西

最早設計建造 **canal lock (near Milan)** 閘門



(2) 歷史概述

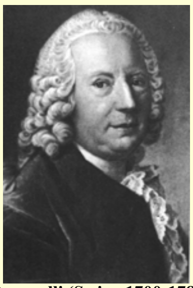
④ **Galileo, Pascal, Newton, Pitot, Bernoulli, Euler, d'Alembert**

Galileo (Italian, 1564 –1642)

Pascal (French, 1623 –1662)

Newton (British, 1642 – 1726)

Pitot (French, 1695 – 1771)



Bernoulli (Swiss, 1700-1782)

Euler (Swiss, 1707-1783)

累積知識 理論←實驗

d'Alembert (French, 1717-1783)

理論與實驗之矛盾與衝突→ **(1) Hydrodynamics** 流體動力學 (偏重數學)

(2) Hydraulics 水力學 (偏重實際應用)

⑤ **Navier-Stokes Equation (19世紀中葉)**

ideal fluid motion → real fluid motion (viscous fluid) 考慮流體之黏滯性

Navier (French, 1785 – 1836)

Stokes (British, 1819 – 1903)

⑥ 同一時期很多 **Hydraulic Research → Empirical formulas** (經驗公式)

or Tabular form (圖表形式)

Physical facts × Empirical formula

⑦ **19**世紀末：航空工業 → 空氣動力學

流力長足進步，由於 **(1) Reynolds** 紊流理論

(2) Rayleigh 因次分析

(3) Froude, Reynolds ... 模型相似律

(4) 航空學 **(aeronautics)** 理論實驗

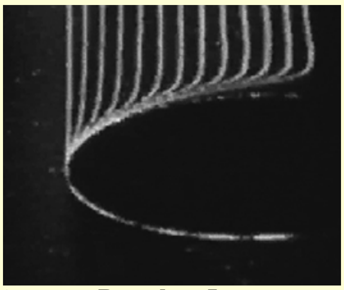
長足進步 **Kutta, Prandtl** 等

Reynolds (British, 1842–1912)

Rayleigh (British, 1842 – 1919)

Froude (French, 1810 – 1879)

Kutta (German, 1867–1944)



Boundary Layer

Prandtl (German, 1875–1953)

⑧流體力學上最重要之貢獻：**Prandtl (1904) → Boundary Layer** (邊界層)

⑨ **Application of fundamental laws of**

<i> Mechanics (力學)

<ii> Thermodynamics (熱力學)

<iii> Experimentation and Simulation (實驗)

<iv> Fluid properties (流體基本性質)

成為⇨ **Modern Fluid Mechanics**

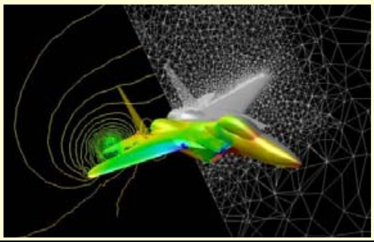
⑩ **21**世紀：流體力學是 **<i>** 一種科學

<ii> 一種工程

以合理**(Rational)**方法尋求答案

高速電腦發展 → 計算流體動力學 **(CFD, Computational Fluid Dynamics)**

更快、更大量之模擬與計算



2. Fluid Properties

(1) Fluid (流體)包括 <i> Liquid (液體)

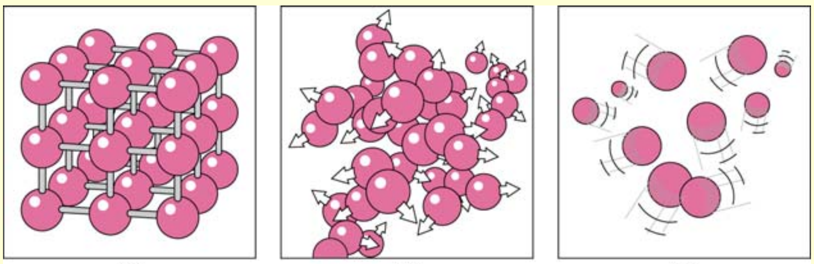
<ii> Gas (氣體)

① 分子間距 **(Spacing of molecules)**

氣體＞液體＞固體

② 分子間內聚力 **(Intermolecular cohesive forces)**

固體＞液體＞氣體



(a) (b) (c)

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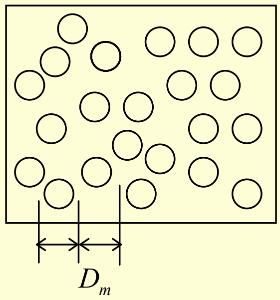
③ **Fluid is a continuum** (假設流體是一種 連體：連續性物質)

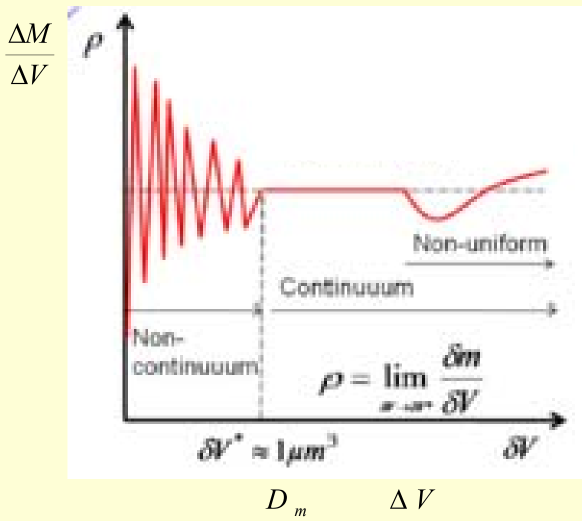
⇨**No voids or holes in continuum.** (連體中沒有空隙)

Note: Continuum assumption is valid only when:

physical length scale＞＞ average molecule spacing



i.e. $\rho = \lim_{\Delta V \to V_*} \frac{\Delta M}{\Delta V}$ $\qquad V_* >> D_m$ (Molecule spacing)

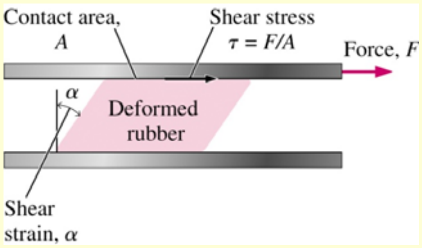


※亦即不視流體之分子結構，而以流體整體之運動為探討對象。(巨觀而非微觀)

④ 物質在應力作用下之反應：**(Actions under various stress)**

<i> 固體承受壓力**(compression)**、張力**(tension)**或剪力**(shear)**，先產生彈性變形**(elastic deformation)**，若外力過大，便產生永久形變**(permanent distortion)**。

<ii> 流體承受壓力時會產生彈性變形。流體幾乎不能承受任何張力，而且在剪力作用下必產生永久形變。



⑤流體**(Fluid)**之正式定義：係一種「不論承受多小之剪力作用，皆會產生持續永久變形」之物質。

⑥ 剪力**(shear)**→帶動流體流動**(flow)**或永久變形

※ 流體靜止**(at rest)**時，剪應力不存在 → 只有壓力**(pressure)**

(2) Fluid Properties

① **Density (密度)**， ρ

$$\rho = \lim_{\Delta V \to 0^+} \frac{\Delta M}{\Delta V}$$

② **Specific Volume (比體積)**， α

$$\alpha = \frac{\Delta V}{\Delta M} = \frac{1}{\rho}$$

③ **Specific Gravity (比重)**， $s.g.$

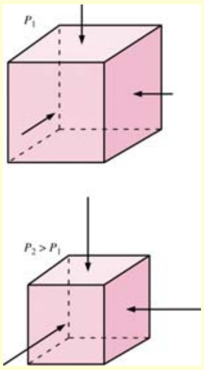
$$s.g. = \frac{\rho}{\rho_{water}}$$ **for specified temperature and pressure (定溫定壓)**

④ **Specific Weight (比重量)**，γ

$$\gamma = \rho g$$

⑤ **Compressibility (可壓縮性)**

If $Temp. = const.$ $P\uparrow, V\downarrow$，$P\downarrow, V\uparrow$



$$dP = -Ev\left(\frac{dV}{V}\right)$$

Ev: bulk modulus of elasticity (彈性模數)

例如： $Ev_{water} = 2.2\times 10^9\ N/m^2$

$1\times 10^6\ N/m^2$ (10 atm) → 體積僅改變0.04% (水之可壓縮性很小，一般可忽略)

For fix amount of fluid: $M = \rho V \rightarrow dM = \rho dV + V d\rho = 0$

$$\frac{d\rho}{\rho} = -\frac{dV}{V} \rightarrow dP = -Ev\left(\frac{dV}{V}\right) = Ev\left(\frac{d\rho}{\rho}\right)$$

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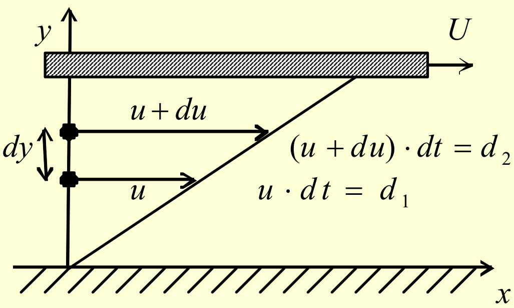
※⑥ **Viscosity (黏滯性) 重要！**

流體黏滯性之成因：

黏性 (液體)：動量傳遞 依靠 分子間之 內聚吸引力 **(cohesion)**
滯性 (氣體)：動量傳遞 依靠 分子間之 碰撞 **(collision)**

No-Slip condition (不滑動條件)



No-Slip condition (不滑動條件)

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① Strain (應變) $= \dfrac{d_2 - d_1}{dy} = \dfrac{du \cdot dt}{dy}$

② shear stress $\tau_{yx} \propto$ Rate of Strain

$$\text{Velocity gradient} = \text{Rate of Strain} = \frac{\text{Strain}}{dt} = \frac{(du \cdot dt / dy)}{dt} = \frac{du}{dy}$$

$$\rightarrow \tau_{yx} \propto \frac{du}{dy} \qquad \boxed{\therefore \tau_{yx} = \mu \frac{du}{dy}}$$

此稱之為 Newton's Viscosity Law

符合 牛頓黏滯律 之流體稱為 牛頓流體 (Newtonian fluid)

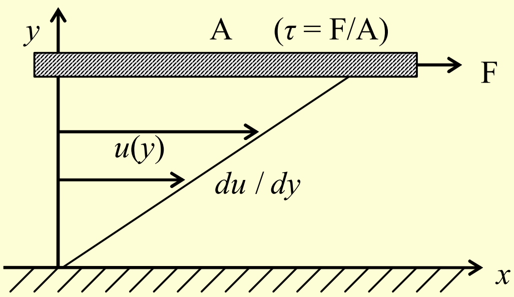
μ = coefficient of viscosity (黏滯性係數)   (單位： $Pa \cdot s$)

= dynamic viscosity (or absolute viscosity)

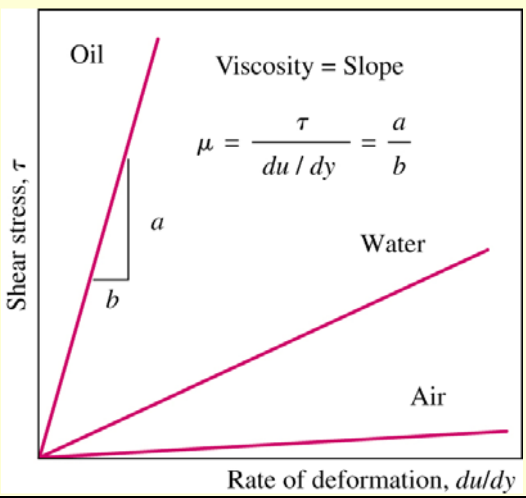
• Kinematic viscosity (運動黏滯性) $\nu = \dfrac{\mu}{\rho}$

Determined by experiments

Newtonian fluid: $\tau_{yx} = \mu \dfrac{du}{dy}$



x



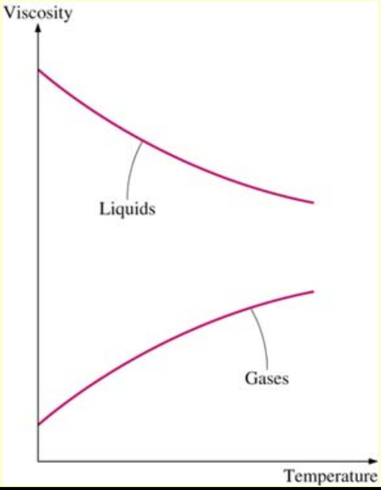
◎ 溫度對 μ 之影響：

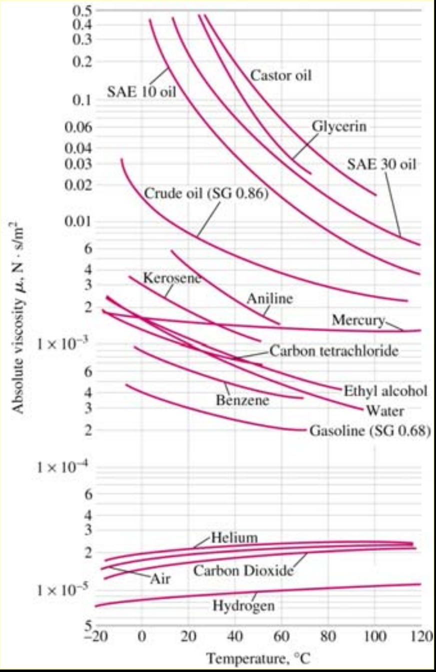
① 液體viscosity → 主要由於分子間內聚力

　氣體viscosity → 主要由於分子間碰撞

② 溫度上升 → 液體分子間內聚力↓ ⇒ μ ↓

　溫度上升 → 氣體分子間碰撞↑ ⇒ μ ↑

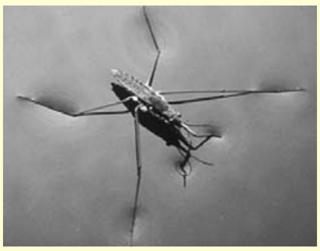


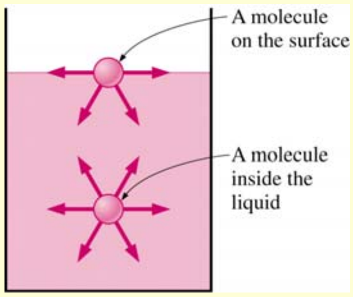


⑦**Surface tension** (表面張力)

(1) 成因： ① **Intermolecular Cohesion** (分子間內聚力)

② **Intermolecular Adhesive Force** (分子間吸附力)





(2) Negligible in many engineering problems.

→**Predominant in** ① **capillary rise of liquid in narrow space**

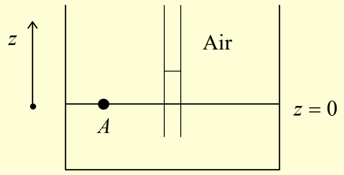
② **bubble formation,　liquid drop.**

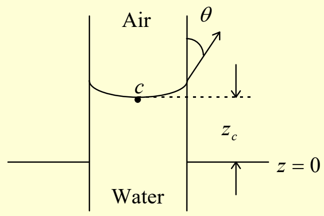
③ **breakup of liquid jets.**

④ **small scale model**

→工程上：地下水 (土壤顆粒間之水分、污染物)

圓形毛細管上升高度 Z_c





$$h = \frac{P}{\rho g} + z$$

$h =$ **piezometric head (or hydraulic head)**

Static: A: $h_A = 0 + 0 = 0$

C: $h_C = \frac{P_c}{\rho g} + Z_c = 0$ **(no flow)**

$\therefore \frac{P_c}{\rho g} = -Z_c$ ※**pressure in capillary is negative!!**

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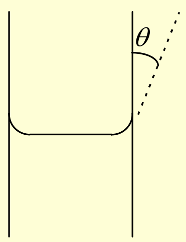
$\sigma_{aw} =$ **Tension force per unit length**

$$\sum F_Z = 0 : \left(\pi r^2\right)\cdot P_c + \sigma_{aw}\cdot\left(2\pi r\right)\cos\theta = 0$$

$$\left(\pi r^2\right)\cdot\left(-z_c\cdot\rho g\right) + \sigma_{aw}\cdot\left(2\pi r\right)\cos\theta = 0$$

$$z_c = \frac{2\sigma_{aw}\cdot\cos\theta}{\rho g\cdot r}$$



通常 $\theta \to 0^\circ$

$\cos\theta \to 1$ $\therefore z_c \cong \frac{2\sigma_{aw}}{\rho g\cdot r}$

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⑧Vapor Pressure (蒸汽壓) 液體表面分子掙脫，分子撞擊形成蒸汽壓

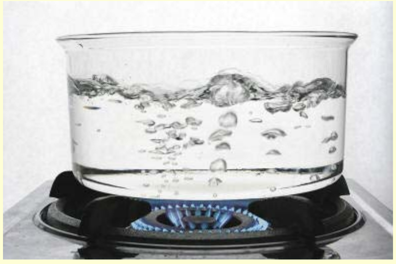
Temp. ↑，Vapor pressure (P_v) ↑

What is Boiling? (何謂沸騰?)

When external absolute pressure (外在絕對壓力) imposed on liquid is equal or less than the vapor pressure of the liquid at that temperature

→ Formation of vapor bubbles

→ Boiling!!



◎ Cavitation (穴蝕)：

Bernoulli's theory tells us:

When velocity ↑ ⇨ External pressure ↓ ⇨ Bubbles form ⇨ Cavitation

⇨ 氣泡高速撞擊渦輪機葉面
或磨蝕管壁，造成損毀！